

THE SIMULATION OF GAS TURBINES BY A STATE  
OF THE ART ANALOG DEVICE

William M. Sheperd

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by

William McMichael Shepherd

Submitted to the Department of Ocean Engineering on  
12 May 1978 in partial fulfillment of the requirements for  
the Degree of Ocean Engineer and the Degree of Master of  
Science in Mechanical Engineering.

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A simulator was designed to model the operation of a  
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A hard-wired desk-top machine was fabricated to simulate  
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Thesis Supervisor: Henry M. Paynter  
Title: Professor of Mechanical Engineering

Thesis Reader: A. Douglas Carmichael  
Title: Professor of Power Engineering

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## I. INTRODUCTION

### A. BACKGROUND

This paper describes the concept and construction of a hard wired analog device, pictured below, which simulates the operation of a gas turbine engine.



Recent work on gas turbine simulation has focused largely on digital or hybrid computing. To accommodate the nonlinear elements of a gas turbine simulation in this fashion requires a perturbation or piecewise linear technique. A pure analog approach offers a continuous



solution to these nonlinear characteristics, which facilitates a simulation in real time.

At this time, analog computation of complex nonlinear systems is not feasible on devices such as the EAI 680 computer currently installed in the M.I.T. Joint Computer Facility. With the miniaturization of both digital and linear functions into the now-common integrated circuit or IC chip, it becomes possible to design extremely compact analog devices. The IC component cost of such a simulation is rapidly approaching the cost of purely passive electronic elements.

The modeling approach to the analog simulation, physical modeling, follows the work of Markunas<sup>1</sup> closely. Physical modeling attempts to explain the engine characteristics within a framework of physical principles and laws which describe the functions occurring in the real machine. Simplicity is generally a strong point of the model.

The model which has been implemented in the analog simulation represents a general machine "invented" by Markunas, who estimated the necessary characteristic relations between the independent variables. Markunas conducted digital and hybrid simulations of his "general" gas turbine, but a full analog implementation was not investigated.



This paper is essentially a design manual which traces the development of a hard-wired analog simulator. It is intended to give the reader a brief overview of the analog devices currently available, and their application to a computational circuit. To facilitate future work with the simulator, a detailed user's manual has been written.

This document contains explicit details relating to the construction and operation of the simulator. The user's manual, the simulator, and associated hardware are under the supervisor of Professor Henry M. Paynter, Department of Mechanical Engineering.

B. DESIGN CONCEPT

The following characteristics were desired in the hard-wired simulator:

1. Real time, non-linear third order simulation of the dynamics of a single shaft gas turbine engine, from start-up to overload conditions.
2. Cost to be under \$600.
3. Size of the device such as to be readily portable and convenient to set-up and operate.
4. Layout and operation of the simulator to be comparable with typical industrial





and military engine control consoles.

5. Accuracy of the steady-state simulation to be considered of secondary importance compared with the representation of large scale transient effects.
6. The ability to investigate various fuel control schemes in a closed loop control, and the ability to operate with different torque-speed load characteristics.
7. Utilization of the general purpose, low cost IC operational amplifier in a hard wired configuration to accomplish all mathematical operations.



## II. MODELING

### A. GENERAL APPROACH

The gas turbine model selected by Markunas will be briefly explained. The modeling approach is broken down into: steady state and dimensional considerations, frequency analysis, and dynamic equations.

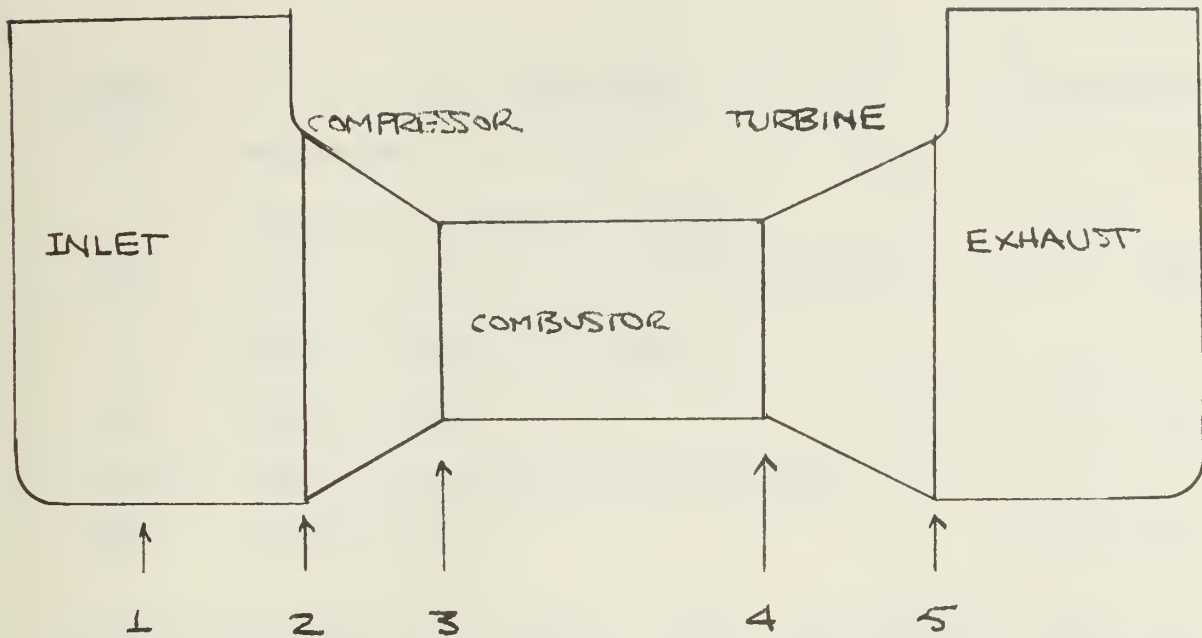
Markunas' estimated performance parameters and the dynamic equations are combined in state-variable format. These equations appear in Appendix II and form the basis for the analog signal flow used in the design.

### B. STEADY STATE AND DIMENSIONAL ANALYSIS

Horlock<sup>3</sup> establishes non-dimensional functions in describing the performance of compressors and turbines. A set of relations are developed to describe the performance of a turbomachine in its simplest terms.

In the subsequent development, gas turbine state points are used, numbered in the following convention, with total values indicated by a zero in front of the state point:





The following symbols are used for dimensional analysis in the compressor:

<u>Symbol</u>	<u>Compressor</u>	<u>Dimensions</u>
$P_{02}$	Inlet stagnation pressure	$M/LT^2$
$\rho_{02}$	Inlet stagnation density	$M/L^3$
$P_{03}$	Outlet stagnation pressure	$M/LT^2$
$T_{02}$	Inlet stagnation temperature	$\theta$
$\eta$	Efficiency	—
$D$	A characteristic linear dimension	$L$
$\Delta T_0$	Stagnation temperature rise through compressor	$\theta$
$N$	Rotational speed	$1/T$
$\dot{m}$	Mass flow rate of gas	$M/T$



<u>Symbol</u>	<u>Compressor</u>	<u>Dimensions</u>
$M$	Molecular wt of gas	—
$\mu$	Absolute viscosity of gas	$M/LT$
$C_p$	Specific heat at constant pressure	$L^2/T^2\theta$
$\gamma$	Ratio of specific heats	—
$R$	Gas constant	$L^2/T^2\theta$
$q_{02}$	Speed of sound of entering gas	$L/T$
$T_1$	Torque input	$ML^2/T^2$

In the compressor, if  $P_{03}$ ,  $\eta$ ,  $T_1$  and  $\Delta T_0$  are chosen as variables of interest, one can write

$$P_{03}, \eta, T_1, \Delta T_0 = f(P_{02}, T_{02}, N, D, \dot{m}, \rho_{02}, \gamma, \mu, q_{02})$$

Assuming a perfect gas, fixing the gas constant,  $R$ , choosing  $\gamma$ , and  $T_{02}$  thus constrains  $q_{02}$ .  $\rho_{02}$  can be expressed in terms of  $R, P_{02}, T_{02}$ ; the original relation becomes

$$P_{03}, \eta, T_1, \Delta T_0 = f(P_{02}, T_{02}, N, D, \dot{m}, R, \gamma, \mu)$$

Taking non-dimensional groups,

$$\frac{P_{03}}{P_{02}}, \eta, \frac{T_1}{P_{02} D^3}, \frac{\Delta T_0}{T_{02}} = f\left(\frac{ND}{\sqrt{T_{02}}}, \frac{\dot{m} \sqrt{RT_{02}}}{P_{02} D^2}, \frac{P_{02} D}{\mu \sqrt{T_{02}}}, \gamma\right)$$

With constant values,  $R$  and  $\gamma$  can be eliminated:

$$\frac{P_{03}}{P_{02}}, \eta, \frac{T_1}{P_{02} D^3}, \frac{\Delta T_0}{T_{02}} = f\left(\frac{ND}{\sqrt{T_{02}}}, \frac{\dot{m} \sqrt{T_{02}}}{P_{02} D^2}, \frac{P_{02} D}{\mu \sqrt{T_{02}}}\right)$$

Multiplying the last two non-dimensional parameters to create a new parameter,





$$\frac{P_{03}}{P_{02}}, \eta, \frac{T_1}{P_{02} D^3}, \frac{\Delta T_0}{T_{02}} = f \left( \frac{ND}{\sqrt{T_{02}}}, \frac{\dot{m} \sqrt{T_{02}}}{P_{02} D^2}, \frac{\dot{m}}{\mu D} \right)$$

It can be seen that the third parameter,  $\frac{\dot{m}}{\mu D}$  is a "flow Reynolds number".

Assuming that the compressor operates above some critical Reynolds number, and that the dimensions of a particular machine are fixed, the equations reduce to:

$$\frac{P_{03}}{P_{02}}, \eta, \frac{T_1}{P_{02}}, \frac{\Delta T_0}{T_{02}} = f \left( \frac{N}{\sqrt{T_{02}}}, \frac{\dot{m} \sqrt{T_{02}}}{P_{02}} \right)$$

In order to present meaningful performance relations, compressor data should be corrected following the above parameters.

Identically for a turbine,<sup>4</sup> it can be shown that the same relations hold and that the parameters of interest in turbine performance<sup>3</sup> are  $\dot{m}, \eta, T_2, \Delta T_0$ . All inlet conditions assume values at state point 4, and all exit values are at state point 5. Torque across the turbine is  $T_2$ . As shown before,

$$\frac{\dot{m} \sqrt{T_{04}}}{P_{04}}, \eta, \frac{T_2}{P_{04}}, \frac{\Delta T_0}{T_{04}} = f \left( \frac{N}{\sqrt{T_{04}}}, \frac{P_{05}}{P_{04}} \right)$$

The characteristics for a given turbomachine can now be specified as functions of dimensionless or pseudo-dimensionless groups:



Compressor:

$$\frac{\dot{m}\sqrt{T_{02}}}{P_{02}} = f\left(\frac{P_{03}}{P_{02}}, \frac{N}{\sqrt{T_{02}}}\right)$$

$$\frac{T_{03}}{T_{02}} = f\left(\frac{P_{03}}{P_{02}}, \frac{N}{\sqrt{T_{02}}}\right)$$

$$\frac{\tau_L}{P_{02}} = f\left(\frac{P_{03}}{P_{02}}, \frac{N}{\sqrt{T_{02}}}\right)$$

Turbine:

$$\frac{\dot{m}\sqrt{T_{04}}}{P_{04}} = f\left(\frac{P_{05}}{P_{04}}, \frac{N}{\sqrt{T_{04}}}\right)$$

$$\frac{T_{05}}{T_{04}} = f\left(\frac{P_{05}}{P_{04}}, \frac{N}{\sqrt{T_{04}}}\right)$$

$$\frac{\tau_T}{P_{04}} = f\left(\frac{P_{05}}{P_{04}}, \frac{N}{\sqrt{T_{04}}}\right)$$



C. FREQUENCY ANALYSIS

Modeling of the processes which take place in a gas turbine can generally be classified as mechanical, fluid, or thermal in nature. To reduce the scope of the problem to a more manageable field, only the most significant energy domains and dynamics were considered. Markunas defines three dynamic regimes for the gas turbine:

1. Low frequency, ( $f \approx .1$  hz) where rotary inertias and thermal capacitances dominate the dynamics.
2. Medium frequency ( $f \approx 1$  hz) where rotary inertias and fluid capacitances dominate.
3. High frequency ( $f \approx 100$  hz) where the fluid dynamics are dominant.

Markunas established that rotor dynamics and fluid capacitances were the primary influences on gas turbine dynamics. Thermal capacitances and fluid inertias were neglected.

D. DYNAMIC EQUATIONS

1. Assumptions

The gas turbine model is assumed to operate with a perfect gas in one-dimensional flow. Components are treated in lumped parameter fashion.



## 2. Rotor Inertia

For the rotor, a summation of torques give the following:

$$\dot{N}_G = \frac{30}{\pi I_R} \sum T$$

where

$\dot{N}_G$  = angular acceleration, rpm/sec

$I_R$  = rotary inertia, ft-lbf-sec<sup>2</sup>

$T$  = load, compressor, or turbine torque, ft-lbf

## 3. Fluid Capacitance

For the one-dimensional lumped parameter fluid, the following symbology is used:

$\rho$  = fluid density, lbm/ft<sup>3</sup>

$x$  = axial distance along duct, ft

$V_x$  = mean axial velocity of fluid, ft/sec

$\gamma$  = ratio of specific heats =  $C_P/C_V$

$C_P$  = specific heat at constant pressure

$C_V$  = specific heat at constant volume

$M$  = Mach number =  $V_x/a$

$a$  = Local sonic velocity =  $\sqrt{\gamma C_V R T}$

$u$  = internal energy of the fluid in ft-lbf/lbm

$h$  = enthalpy of the fluid in ft-lbf/lbm

$VOL$  = lumped fluid volume





A general mass conservation equation for the one dimensional fluid stream may be written:

$$\frac{\partial(\rho)}{\partial t} + \frac{\partial}{\partial x}(\rho V_x) = 0$$

Integrating over a constant area duct, with fluid properties constant at a given axial distance or dimension:

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho A V_x) = 0$$

$$\frac{\partial A}{\partial x} \equiv 0 \quad \text{by definition.}$$

Integrating over the length of the duct

$$\frac{\partial}{\partial t} \int_L \rho A dx + \int_L \frac{\partial}{\partial x}(\rho A V_x) dx = 0$$

$$\frac{\partial}{\partial t}(\bar{\rho} \cdot \text{vol}) + \rho A V_x \Big|_0^L = 0$$

$\int_L \rho A dx$  is approximated by an average density,  $\bar{\rho}$ . Lumped fluid volume (vol) since

$$\frac{d}{dt}(\bar{\rho}) = \frac{1}{\text{vol}}(\dot{m}_1 - \dot{m}_2)$$



where the subscripts 1 and 2 identify inlet and outlet conditions across the engine, respectively.

Writing a similar equation for the conservation of momentum neglecting viscous shear and body forces,

$$\frac{\partial}{\partial t} \left( \frac{\rho V_x}{g_c} \right) + \frac{\partial}{\partial x} \left( \frac{\rho V_x^2}{g_c} + P \right) = 0$$

Integrating over area,

$$\frac{\partial}{\partial t} \left( \frac{\rho V_x A}{g_c} \right) + \frac{\partial}{\partial x} \left( \frac{\rho V_x^2 A}{g_c} + PA \right) = 0$$

$$\frac{\partial}{\partial t} \left( \frac{\rho V_x A}{g_c} \right) + \frac{\partial}{\partial x} \left( \frac{\rho V_x^2 A}{g_c} \right) + \frac{\partial}{\partial x} (PA) = 0$$

$$\frac{\partial}{\partial t} \left( \frac{\rho V_x A}{g_c} \right) + \frac{V_x^2 A}{g_c} \frac{\partial \rho}{\partial x} + A \frac{\partial}{\partial x} (P) = 0$$

Integrating over length of the duct

$$\frac{\partial}{\partial t} \int_L \left( \frac{\rho V_x A}{g_c} \right) dx + \left. \frac{\rho V_x^2 A}{g_c} \right|_1^2 + \left. PA \right|_1^2 = 0$$

$$\frac{\partial}{\partial t} \int_L \frac{\dot{m}}{g_c} dx + \left[ \frac{\rho V_x^2 A}{g_c} + PA \right]_1^2 = 0$$

$$\frac{d}{dt} \left( \frac{\dot{m} L}{g_c} \right) = \left[ PA + \frac{\rho V_x^2 A}{g_c} \right]_2^1$$

$$\frac{d}{dt} (\bar{\dot{m}}) = \frac{g_c A}{L} \left[ P + \frac{\rho V_x^2}{g_c} \right]_2^1$$

$$\bar{\dot{m}} \equiv \text{AVERAGE VALUE}$$



$$M^2 = \frac{V_x^2}{a^2} = \frac{V_x^2}{g_c \gamma R T}$$

$$\frac{d}{dt}(\bar{m}) = \left[ \frac{g_c A}{L} \right] \left[ P(1 + \gamma M^2) \right]_2^1$$

Since total pressure,  $P_0 = P(1 + \frac{\gamma-1}{2} M^2)^{\frac{\gamma}{\gamma-1}}$

$$\frac{d}{dt}(\bar{m}) = \left[ \frac{g_c A}{L} \right] \left[ \frac{P_0(1 + \gamma M^2)}{(1 + \frac{\gamma-1}{2} M^2)^{\frac{\gamma}{\gamma-1}}} \right]_2^1$$

Assuming  $\gamma = 1.4$ , and  $M \leq .5$ , the momentum equation can be written:

$$\frac{d}{dt}(\bar{m}) \approx \frac{g_c A}{L} (P_{01} - P_{02})$$

Similarly for energy conservation with no external work or heat addition,

$$\frac{\partial}{\partial t} \left[ \rho \left( u + \frac{V_x^2}{2g_c} \right) \right] + \frac{\partial}{\partial x} \left[ \rho V_x \left( h + \frac{V_x^2}{2g_c} \right) \right] = 0$$

Integrating over area and length,

$$\frac{\partial}{\partial t} \int_0^L \left[ \rho A \left( u + \frac{V_x^2}{2g_c} \right) \right] dx + \bar{m} \left( h + \frac{V_x^2}{2g_c} \right) \Big|_1^2 = 0$$



$$(\text{VOL}) \frac{d}{dt} \left[ \rho \left( u + \frac{V_x^2}{2g_c} \right) \right] = \dot{m} h_o \Big|_2^1$$

where

$$h_o \equiv h + \frac{V_x^2}{2g_c}$$

Since

$$\frac{V_x^2}{2g_c} = \frac{\gamma R T M^2}{2}$$

Substituting,

$$u + \frac{V_x^2}{2g_c} = c_v T \left( 1 + \frac{\gamma(\gamma-1)}{2} M^2 \right)$$

$$\frac{d}{dt} \left[ \left( \bar{\rho} c_v T_o \right) \frac{1 + \frac{\gamma(\gamma-1)}{2} M^2}{1 + \frac{\gamma-1}{2} M^2} \right] = \frac{C_p}{\text{VOL}} (\dot{m}_1 T_{o1} - \dot{m}_2 T_{o2})$$

where  $T_o = T + \left( 1 + \frac{\gamma-1}{2} M^2 \right)$

With  $M \leq .5$  the result can be simplified to:

$$\frac{d}{dt} (\bar{\rho} T_o) = \frac{\gamma}{\text{VOL}} (\dot{m}_1 T_{o1} - \dot{m}_2 T_{o2})$$

#### E. STATE EQUATIONS

The three conservation equations are now:

1. Mass

$$\frac{d}{dt} (\bar{\rho}) = \frac{1}{\text{VOL}} (\dot{m}_1 - \dot{m}_2)$$





2. Momentum

$$\frac{d}{dt}(\bar{m}) = \frac{\gamma c A}{L} (P_{01} - P_{02})$$

3. Energy

$$\frac{d}{dt}(\bar{P}T_0) = \frac{\gamma}{\text{vol}} (\dot{m}_1 T_{01} - \dot{m}_2 T_{02})$$

Using the definition of a polytropic process,

$PV^n = \text{CONST}$ , it follows that

$$P_0 = (\text{CONST}) \rho_0^n$$

$$T_0 = (\text{CONST}) P_0^{\frac{n-1}{n}} = (\text{CONST}) \rho_0^{n-1}$$

In order to get the mass and energy equations into more convenient form,

$$\frac{d}{dt}(P_0) = (\text{CONST})(n) \frac{d}{dt}(\rho_0)$$

$$\frac{d}{dt}(\rho_0) = \left( \frac{1}{nRT_0} \right) \frac{d}{dt}(P_0)$$

$$\frac{d}{dt}(\rho_0 T_0) = \text{CONST} \rho_0^n \frac{dT_0}{dt} + (n-1) \text{CONST} \rho_0 \frac{d\rho_0}{dt} T_0$$

$$\frac{d}{dt}(\rho_0 T_0) = \left( \frac{n}{n-1} \right) \left( \frac{P_0}{RT_0} \right) \frac{d}{dt}(T_0)$$

( WHERE  $\bar{T}$  IS ASSUMED TO TAKE STAGNATION VALUES )



With the result that the equations now be more:

$$\frac{d}{dt}(\bar{P}_0) = \frac{\bar{nRT}_0}{VOL} (\dot{m}_1 - \dot{m}_2)$$

$$\frac{d}{dt}(\dot{m}) = \frac{\bar{g_c A}}{L} (\bar{P}_{01} - \bar{P}_{02})$$

$$\frac{d}{dt}(\bar{T}_0) = \frac{n-1}{n} \left( \frac{C_p R \bar{T}_0}{C_v \bar{P}_0 VOL} \right) (\dot{m}_1 \bar{T}_{01} - \dot{m}_2 \bar{T}_{02})$$

Where 1 and 2 represent inlet and outlet across the duct. Because of lumped parameter considerations, machine model parameters can be substituted as follows:

One Dimensional

$\dot{m}_1$

$\dot{m}_2$

$\bar{T}_0$

$\bar{T}_{01}$

$\bar{T}_{02}$

$\bar{P}_0$

Machine Model

$\dot{m}_3$

$\dot{m}_4$

$\bar{T}_{04}$

$\bar{T}_{03}$

$\bar{T}_{04}$

$\bar{P}_{03}$

Substituting notation for the first and third differential equations,

$$\dot{\bar{P}}_{03} = \left( \frac{\bar{nRT}_{04}}{VOL} \right) [\dot{m}_3 - \dot{m}_4]$$



$$\dot{T}_{04} = \left( \frac{(n-1)}{n} \frac{\gamma R T_{04}}{P_{03} \text{ VOL}} \right) \left[ \dot{m}_3 T_{03} + \dot{m}_F \left( \frac{h_{LHV}}{c_p} \right) - \dot{m}_4 T_{04} \right]$$

Introducing the torque equation as the third system equation,

$$\dot{N}_G = \left( \frac{30}{\pi I} \right) \left[ T_2 - T_L - T_3 \right]$$

where

$h_{LHV}$  = fuel lower heating value, Btu/lbm

$\dot{m}_F$  = fuel flow, lbm/sec

$T_3$  = load torque, ft-lbf

$P_{03}$ ,  $T_{04}$ , AND  $N_G$  are chosen as variables of convenience.

For an isentropic process,  $n = 1.4$ . Markunas felt that  $n-1/n$  should be replaced by 1.0 in the temperature equation.

## F. ENGINE DATA, ASSUMPTIONS, AND CHARACTERISTICS

### 1. Introduction

In Markunas' analysis a single shaft machine was invented from a number of sources. Several real world machines exist which have design parameters close to



Markunas's engine. The recent Garrett GTPF 990 engine in its locked shaft mode is a reasonable example.

## 2. Design Point Specifications

Pressure Ratio	5.0
Compressor Isentropic Efficiency	1280°F
Turbine Isentropic Efficiency	88%
Compressor Discharge Mass Flow	100 lbm/sec
Inlet Pressure	14.175 psia
Inlet Temperature	80°F
Rotor Speed	7200 rpm
Output	6948 HP

Markunas plotted characteristic curves for his compressor and turbine, which appear in Appendix I. Markunas then mathematically "fit" each characteristic curve. These approximations follow the functional relations established for non-dimensional and pseudo dimensionless groups.

## 3. Compressor and Turbine Characteristics

Markunas defined the following variables and characteristics for his compressor:

$$PRC = P_{03}/P_{02} \quad \text{pressure ratio}$$





$$TRC = T_{03} / T_{02}$$

temperature ratio

$$XWC = \frac{\dot{m}_3 \sqrt{T_{02}}}{P_{02}} \left/ \left( \frac{\dot{m}_3 \sqrt{T_{02}}}{P_{02}} \right)_{\text{DESIGN}} \right.$$

scaled compressor  
mass flow

$$XNC = \frac{NG}{\sqrt{T_{02}}} \left/ \left( \frac{NG}{\sqrt{T_{02}}} \right)_{\text{DESIGN}} \right.$$

scaled compressor  
speed

$$XMC = \frac{P_{02}}{\sqrt{T_{02}}} \left( \frac{\dot{m}_3 \sqrt{T_{02}}}{P_{02}} \right)_{\text{DESIGN}}$$

$$AC = .75 (XNC) + .27 (XNC)^3$$

$$BC = 3.0 (XNC)^{1.5} + 3.0 (XNC)^7$$

$$CC = .018 (2 - XNC)^{5.8}$$

$$DC = .891 \sqrt{.9 - (XNC - .9)^2}$$

$$EC = 1.0 + XNC + 3.6 (XNC)^{4.1}$$

$$FC = \begin{array}{ll} 4.0 & \text{IF } PRC \leq EC \\ 2.0 & \text{IF } PRC > EC \end{array}$$



$$GEC = \frac{\gamma-1}{\gamma}$$

$$XWC = AC \left[ 1 - \left( \frac{PRC-1}{BC} \right) \right]^{CC}$$

$$EFFC = DC \left[ 1 - \left( \frac{EC-PRC}{EC-1} \right) \right]^{FC}$$

$$TRC = 1 + \left[ \frac{(PRC)^{GEC} - 1}{EFFC} \right]$$

Similarly, for the turbine

$$PRT = P_{05}/P_{04}$$

$$TRT = T_{05}/T_{04}$$

$$XWT = \frac{\dot{m}_4 \sqrt{T_{04}}}{P_{04}} \left/ \left( \frac{\dot{m}_4 \sqrt{T_{04}}}{P_{04 \text{ DESIGN}}} \right) \right.$$

$$XMT = \frac{P_{03}}{\sqrt{T_{04}}} \left( \frac{\dot{m}_4 \sqrt{T_{04}}}{P_{05 \text{ DESIGN}}} \right)$$

$$XNT = \frac{N_G}{\sqrt{T_{04}}} \left/ \left( \frac{N_G}{\sqrt{T_{04 \text{ DESIGN}}}} \right) \right.$$

$$AT = 1.0006$$

$$BT = 3.1 + \left( \frac{1.62}{XNT + 0.26} \right)$$



$$CT = .7$$

$$DT = .88 \left[ 1 - (1 - XNT)^4 \right]^{.65}$$

$$ET = 1 - .8(XNT)^{1.25}$$

$$GET = \frac{8-1}{8}$$

$$XWT = AT \left[ 1 - (PRT)^{BT} \right]^{CT}$$

$$EFFT = 1 - \left[ 1 - \sqrt{\frac{1 - ET}{1 - PRT}} \right]^2$$

$$TRT = 1 - EFFT \left[ 1 - (PRT)^{GET} \right]$$

#### Additional Machine Assumptions

1.  $P_{05} = P_{02} = P_{01}$  (no duct loss)
- $P_{03} = P_{04}$  (no combustor losses)

The machine characteristics are combined with the differential equations to produce a set of system state equations. The expanded equations appear in Appendix II.



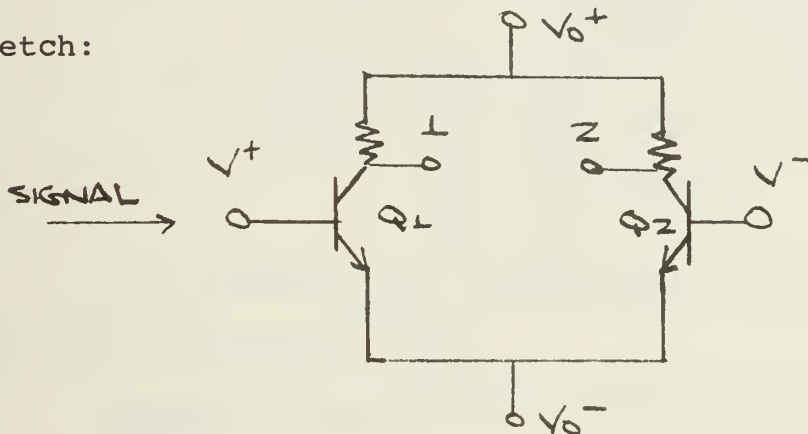
### III. THE OPERATIONAL AMPLIFIER

#### A. INTRODUCTION

The fundamentals of the operational amplifier, (OA) are briefly presented to establish the necessary analog "tools" to proceed to a simulator design.

#### B. BASIC OPERATION

An ideal OP amp can be modeled by the following sketch:



with no differential voltage between positive ( $V^+$ ) and negative ( $V^-$ ) inputs, both emitters are equally biased, and collector currents  $Q_1$  and  $Q_2$  are equal. The differential voltage output between 1 and 2 is also zero. Driving  $V^+$  more positive with a signal will cause the current through  $Q_1$  to increase. The current through  $Q_2$  must decrease by a proportional amount, so that in the differential voltage output between 1 and 2, 1 gets driven more negative with respect to its initial voltage, and 2 becomes more positive.



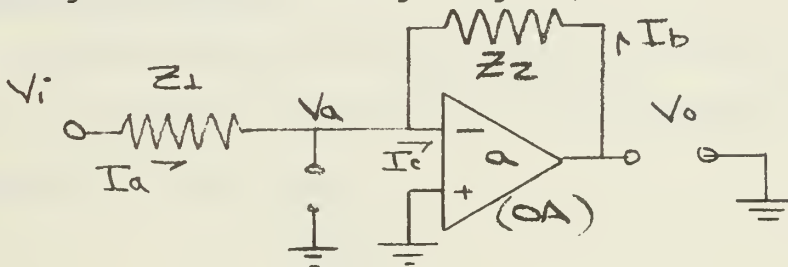


Thus for a small signal applied to one of the input terminals, a high voltage gain and current "buffering" takes place as a result of the transistor characteristics.

With the ideal OA, the input current is essentially zero, and the voltage gain is assumed infinite.

### C. THE INVERTING OA

Looking at the following diagram,



It is assumed that the impedance of the amplifier  $Z_c \rightarrow \infty$ , therefore  $I_c \approx 0$ .

Closed Loop Gain, from a summation of currents:

$$I_b = (V_o - V_a) / Z_2$$

$$I_a = (V_i - V_a) / Z_1 ; I_a \approx -I_b$$

$$-V_o / Z_2 + V_a / Z_2 = V_i / Z_1 - V_a / Z_1$$

$$\frac{V_o}{V_i} = \frac{-Z_2 / (Z_1 + Z_2)}{1 + [Z_1 / (Z_1 + Z_2)]}$$



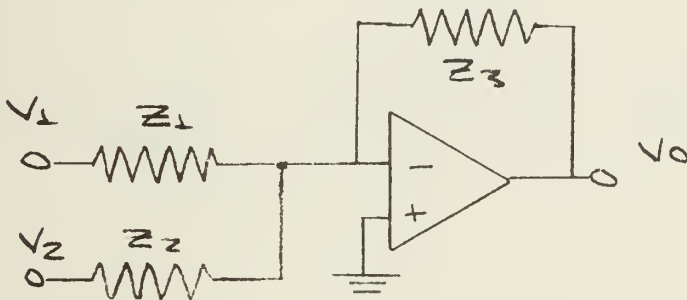
If  $a$ , the amplifier gain, is large,

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

It can therefore be seen that the inverting OA inverts and provides a gain proportional to the input and the signal feedback resistances.

#### D. INVERTING SUMMER

By joining multiple input currents to the input side of the OA, an inverting summer can be constructed with variable gains on each input.



By current summation,

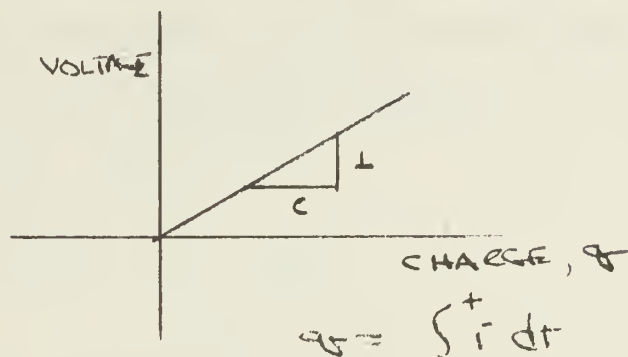
$$V_o / Z_3 = -V_1 / Z_1 - V_2 / Z_2$$

$$V_o = -V_1 \frac{Z_3}{Z_1} - V_2 \frac{Z_3}{Z_2}$$

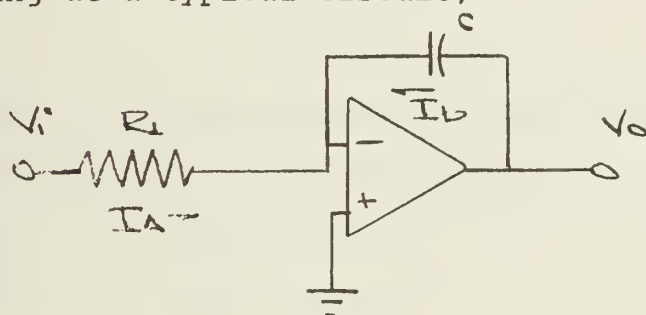


# E. INTEGRATOR

Adding a capacitor to the feed back loop of the OA adds the following constitutive relation to the feedback loop:



Looking at a typical circuit,



$$I_A = -I_B$$

$$\frac{\Delta V_o}{\Delta t} = \frac{I_B}{C}$$

$$\Delta V_o = -\frac{V_i}{R, C} \Delta t$$

$$V_o = -\frac{1}{R, C} \int_0^+ V_i dt$$



# F. LOGARITHMIC GENERATOR

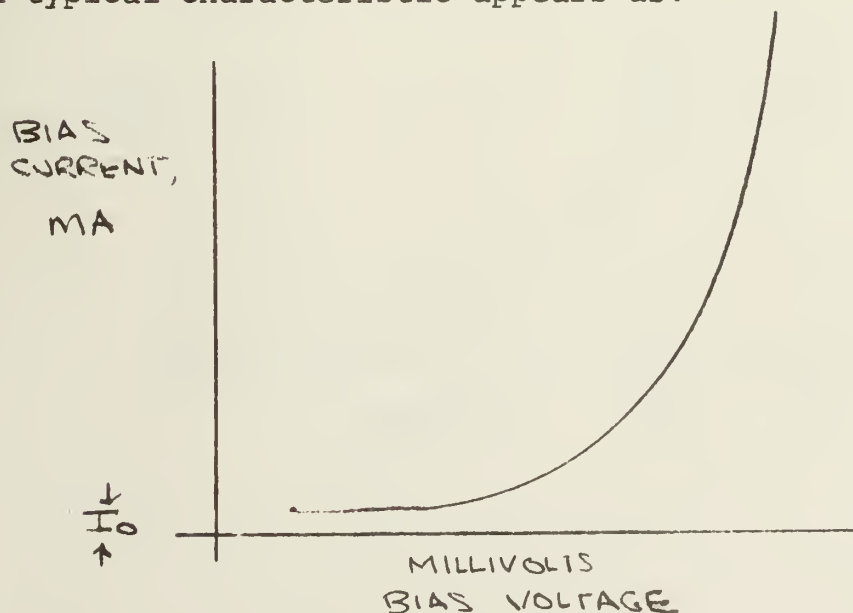
Using a diode in the feedback loop enables the logarithmic nature of the diode voltage - current relation to be included in the loop. The diode characteristic can be modeled by the following equation:

$$\begin{aligned}\frac{I}{I_0} &= \exp\left(\frac{V}{V_0}\right) - 1 \\ \frac{V}{V_0} &= \ln\left[1 + \frac{I}{I_0}\right] \\ V &= V_0 \ln(I_0 + I)\end{aligned}$$

where:

$I_0$  = junction saturation current  
 $V_0$  = junction saturation voltage  
 $I$  = bias current  
 $V$  = bias voltage

A typical characteristic appears as:

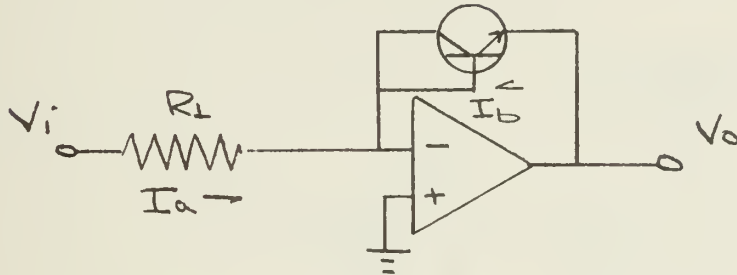




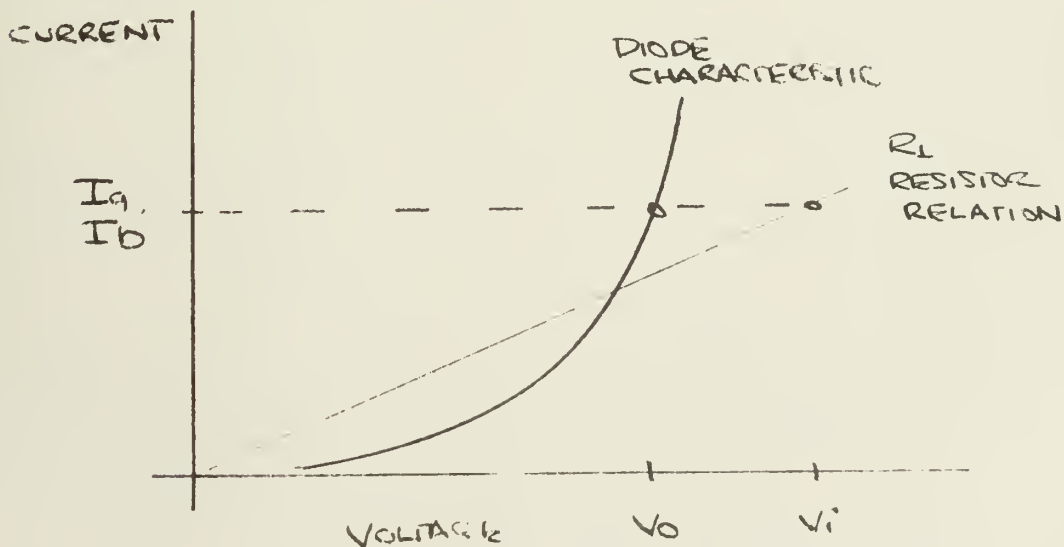


It is important to note that the saturation voltage  $V_0$  is directly proportional to the absolute temperature.

A typical log circuit can be constructed as follows:



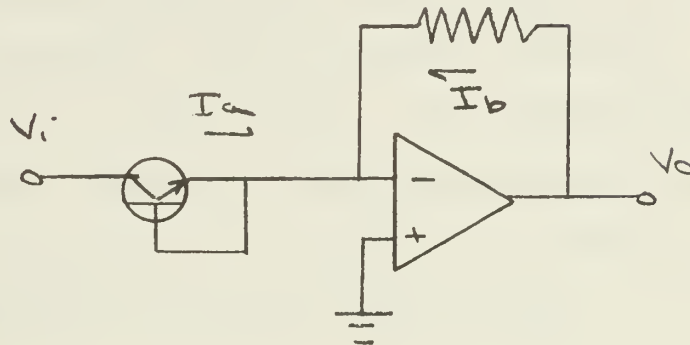
A diode - connected transistor is used to perform the diode function, as transistor emitter-base junctions have more dynamic current range than diodes in general. By the following diagram, an input  $V_i$  across  $R_L$  causes a current flow through the "transdiode". The bias voltage, appearing as  $V_0$ , varies as the log of  $V_i$ .





G. ANTILOG GENERATOR

In a similar fashion, input voltages or currents may be exponentiated in the following circuit:





#### IV. ANALOG COMPUTATION

##### A. INTRODUCTION

To implement the non-linear system equations the following functional operations are performed: addition, subtraction, integration, multiplication, division, exponentiation to a fixed power, and exponentiation to a variable power. The first three analog operations have been briefly discussed. To implement the higher functions, the use of logarithmic circuits must be made.

##### B. THE PRACTICAL LOG CIRCUIT

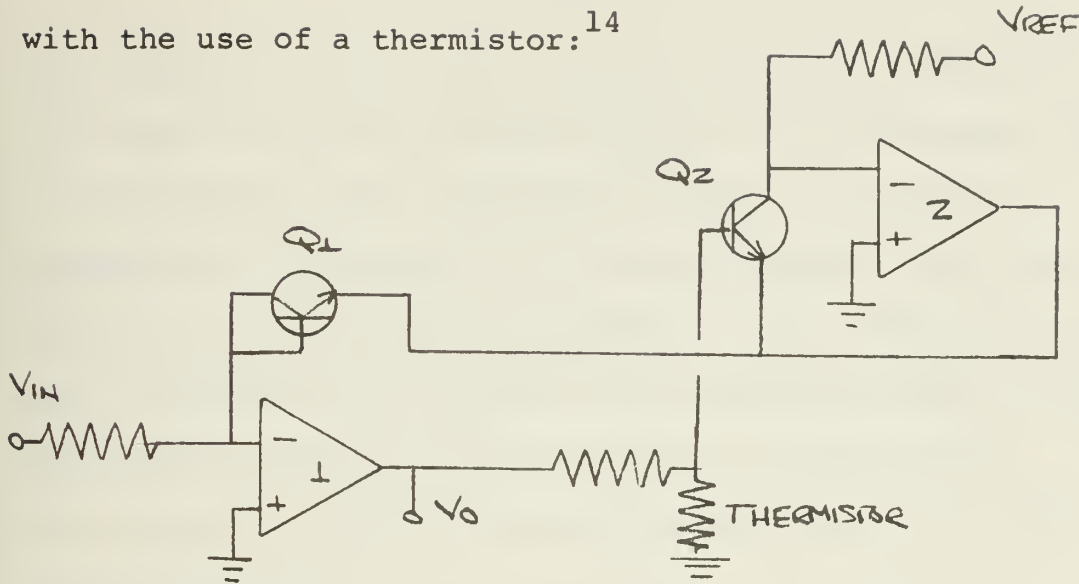
There are at least three major considerations in the construction of a practical log or antilog circuit: cost, complexity, and accuracy. Three different architectures are considered to accomplish the log function.

The first choice is the previously discussed simple log/antilog circuit. Using low quality components, cost of the log circuit is under one dollar. Thermal stability is considered poor. A test circuit and transdiode characteristics are shown in Appendix III. Test data indicated that the accuracy and thermal properties of this circuit were marginal.

A second choice is a compensated log circuit as diagrammed below, which offers increased thermal stability



with the use of a thermistor:<sup>14</sup>



In the diagram, the collector of  $Q_2$  is kept at virtual ground by the OP amp, and the collector current through  $Q_2$  is equal to a constant reference current provided by  $V_{REF}$ . It can be shown that the output voltage is of the form  $V_O = -\log_{10} \left( \frac{I_T}{I_R} \right)$  where the thermistor provides temperature compensation. One of the drawbacks to this scheme is that the number of components necessary to generate the log function has gone from 3 to at least 9, with proportional space and cost increases. This configuration was not evaluated.

The third and most expensive choice is to utilize a dedicated logarithmic device. The particular unit tested was a Texas Instrument log amplifier. This is a dual log generator on a single DIP package, at a cost of

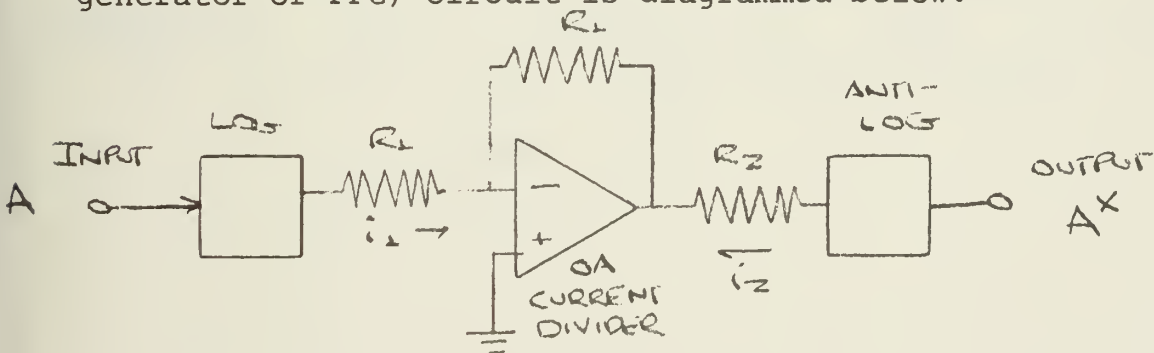




approximately \$2.50. With appropriate circuit manipulation, the desired log/anti-log operations can be performed. The TI log amplifier was evaluated in depth. Results are documented in Appendix III. The log amplifier was the foundation of the simulator design. Other dedicated log/antilog modules exist which offer increased dynamic range, temperature stability, and accuracy, but alternative devices were an order of magnitude more costly than the TI log amplifier.

#### C. EXPONENTIATION - ALL CASES

In order to achieve either fixed or variable power exponentiation, the use of logarithms is basic. The signal flow for a fixed power exponent (power function generator or PFG) circuit is diagrammed below:



By a judicious selection of values for  $R_1$  and  $R_2$  current can be multiplied or divided using an operational amplifier in the ratio:



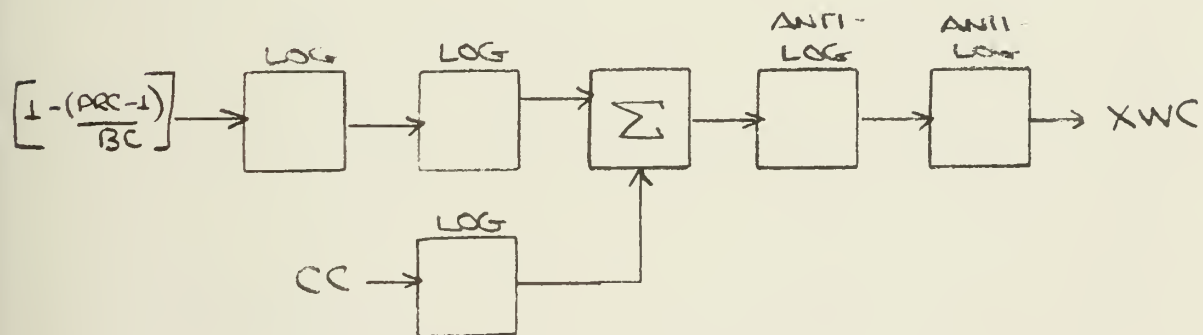
$$\frac{i_2}{i_1} \approx \frac{R_1}{R_2}$$

To be able to exponentiate to a variable power requires a cascaded log/antilog arrangement. The use of a segment of the state equations is made to demonstrate the concept. It is desired to generate the following:

$$XWC = K \left( 1 - \frac{(PRC-1)}{BC} \right)^{CC}$$

where  $K$  represents the scaling of several fixed parameters, and  $XWC$ ,  $PRC$ ,  $BC$  and  $CC$  are all problem variables.

The circuit selected must process:



As a result of thermal instabilities, the interior section of the signal flow was replaced with a four-quadrant multiplier.

The multiplier essentially compresses the separate functions into a single chip. By circuit manipulation,



a multiplier can also divide, square, and take square roots. A range of multiplier devices currently exist, with the price of the least expensive about \$3.

Test data on the multiplier used in the simulator was collected in Appendix III. To allow as compact a circuit as possible, the multiplier was frequently utilized in its dividing and exponentiating modes. Complete device specifications are included in Appendix VI.



## V. PRELIMINARY DESIGN

### A. INTRODUCTION

The simulator design proceeded in a parallel fashion. In an iterative process, estimations were generated for the electronic layout, which allowed the cabinet and control panel to be sized and designed.

The simulator is configured to resemble a real-world engine control panel. Additional concepts for the peripheral design which were not incorporated were:

1. Proportional fuel control schemes with variable gains.
2. Torque-speed loads characteristic of pumps, propellers, generators, and other real world devices.
3. Overspeed alarms and stall audio noise circuits.

### B. ELECTRONICS

#### 1. Logic Circuit

The logic processing of the electronic circuitry was broken down into 3 separate grid structures, which appear as design drawings in Appendix V. Use was made of bread-board panels to avoid the inflexibility of other techniques. Concurrent with testing various mathematical





operations, the circuit design was modified to balance the competing parameters of simplicity, size, accuracy, and cost. The appropriate equation variables were normalized about their design point values, and then scaled to provide a circuit signal that was approximately 75% of the maximum rates input for the device. The steady state design point values for all variables were established by a DYSYS program and used for the normalization. Additional notes on the wiring techniques and breadboard layout are included in Appendix VI.

## 2. DYSYS Simulation and Scaling

A DYSYS simulation of the system equations was run as it appears in Appendix IV. Significant comments on the program are:

1. The conditional value of FC was changed to a fixed value of 3.0 to facilitate the circuit logic.
2. Markunas estimated a parabolic "pump-type" load torque, which appears as  $.0000977(3)**2$ .
3. The modeling parameter DT is called TT in the program to avoid confusion with the DYSYS time-step variables, DT.



The scaling procedure started with the following listings:

a. Constants

$P_{O2}, P_{O5}$	=	2041 LBF/FT <sup>2</sup>
$T_{O2}$	=	539.7°R
$\gamma$	=	1.4
$R$	=	53.4 FT-LBM/LBM°R
$C_p$	=	.291 BTU/LBM-°R
$h_{LHV}$	=	16994 BTU/LBM

b. Machine Characteristics

$$I = 25 \text{ FT}^4$$

$$\left( \frac{N_G}{\sqrt{P_{O2}}} \right)_{\text{DESIGN}} = 304.8$$

$$\left( \frac{N_G}{\sqrt{P_{O1}}} \right)_{\text{DESIGN}} = 171.1$$

$$\left( \frac{\dot{m}_3 \sqrt{P_{O2}}}{P_{O2} \text{ DESIGN}} \right) = 1.138$$

$$\left( \frac{\dot{m}_3 \sqrt{P_{O4}}}{P_{O4} \text{ DESIGN}} \right) = 4086$$



c. Input Variables at Design Point

$$\begin{aligned}\dot{m}_F &= 1.1807 \text{ LBM/SEC} \\ T_3 &= 5028 \text{ R-URF}\end{aligned}$$

d. State Variables at Design Point

$$\begin{aligned}P_3 &= 1027 \text{ LBF/IN}^2 \\ N_3 &= 7200 \text{ RPM} \\ T_0 &= 1731 \text{ R}\end{aligned}$$

The DYSYS simulation was run to verify Markunas's simulations. Use was made of the program to assist in adding voltage and current values to the circuit design, and to evaluate the effect of fixing the discontinuous constant, FC. FC was conditionally assigned the value of 2 or 4 in Markunas work. The steady state effect of assigning FC as 3 was investigated and adopted.

The DYSYS simulation was also run off - design to generate a test case for the logic circuit. The simulation was run at approximately 50% fuel flow and steady-state values were generated for all elements of the equations.



Some difficulty was encountered in determining the correct value for the lumped parameter fluid volume. The value which appeared to be consistent with Markunas' data was 144 ft<sup>3</sup>. An approximation based on real world data gave 15 ft<sup>3</sup> as a more likely figure. Since the volume represents a time constant scaling for the fluid capacitance, it has no significance for the DYSYS steady state values. Changes in the volume assumed can be readily investigated in the simulator dynamic response.

The scaled voltage values and the system variables they represent are drawn on the 3 circuit plans in Appendix V.

#### C. PERIPHERALS

The main elements of the remaining design work included the chasis, power supply and distribution, instrumentation and control hardware, and breadboard architecture. The design drawings and sketches included in Appendix V illustrate the final peripheral design. The photograph below shows the internal layout of the simulator.







Appendix VI, specifications, contains additional data on the peripheral equipment, its wiring, and operating parameters.



## VI. IMPLEMENTATION AND CHECKOUT

### A. PROCEDURE

To establish the characteristics of the various devices and to investigate the methodology of the design, 10 tests were conducted. The significant results are discussed here in the context of the design process. All test data is included in Appendix III.

### B. LOG GENERATION

Tests 1 and 2 determined the voltage - current relations for the transdiode - connected OP amp, using both 2N2222 and 2N2904 transistors. It became apparent that the allowable voltage values of the log circuit were so narrow that only current variations could be used for the signal, over about a 1 1/2 decade range of values.

### C. POWER EXPONENTIATION

Tests 3, 4, 5, and 6 were designed to varify the fixed power exponentiation circuitry. A workable configuration was wired in test 6, using a current - dividing resistive network.

Tests 7, 8, 9, and 10 were conducted to validate the variable - power exponentiation procedure. The inability of the transdiode - connected OP amp to provide a stable



cascaded log circuit led to the investigation of a 4 quadrant multiplier in Test 8.

The Exar 2208 multiplier characteristics were investigated to generate hookup diagrams, trimming values, and allowable voltage values.

With Test 9, a simple log/multiplier/simple antilog circuit was wired and observed. Stability was significantly improved, but was still considered below the accuracy level desired.

In Test 10, a Texas Instrument 441 log amplifier was tested to determine its log/antilog characteristics. The log-amp was incorporated into the variable exponentiation circuit and the stability evaluated. The steady state accuracy of the log amplifier in the variable exponentiation circuit was somewhat better than the simple log configuration. Since the dedicated log amplifier offered better compensation characteristics than the simple log configuration, it was substituted at all points in the circuit design.

#### D. GENERAL CIRCUIT DESIGN

Concurrent with the test sequence, the overall circuit design was changing as new test data was generated. The more significant problems which caused iteration of the circuit are briefly discussed. One immediate limitation



exists using the TI log amplifier module. Looking at "TL 441 Transfer Characteristics" included in Appendix III, the log amplifier only has a useable output range of 0 - 450 millivolts. This can be doubled by wiring an OP amp to the input side of the log amplifier to use the dual input capability of the device. Still, the output accuracy is limited, since various offsets for multipliers and OP amps can easily reach 10 millivolts.

As a general rule, it was decided to trim the circuit at a limited number of points, rather than dealing with the offsets required for each component.

It was intended in the final circuit design to operate all components at one dual-polarity supply voltage. This choice nominally would be  $\pm 8$  volts, the maximum rating of the most sensitive device, the TI log amp. Test 10 demonstrated that significant nonlinearities were introduced into the multiplier characteristics if the supply voltage were reduced. Three solutions were postulated:

1. Run the multipliers at  $\pm 15$  volts, and the OP and log amps at  $\pm 8$  volts.
2. Compensate the multiplier to have linear characteristics at  $\pm 8$  volts supply.
3. Replace the multiplier modules with log amplifiers to process the same function.





The first choice was selected in an effort to keep the circuit design as simple as possible, in as much as the single - voltage supply was more of a packaging consideration than an electronic one.



## VII. CONCLUSIONS AND RECOMMENDATIONS

### A. FEASIBILITY OF THE DESIGN

From the test data collected, a real time simulation can be conducted with the logic circuits wired as designed. The techniques developed for the implementation of non-linear differential equations should be applicable to a wide range of system models.

Markunas defined his engine in essentially non dimensional or pseudo-dimensionless groups. It was not possible to investigate the ability of the simulator to represent different geometric machines, however, it is felt that future work will verify the utility of the simulator and the nonlinear model.

### B. DESIGN REFINEMENTS

The simulator as designed could be significantly enhanced with more general torque and fuel control schemes. If the simulator could demonstrate sufficient generality in modeling the behavior of real world gas turbines, an autonomous "observer" circuit could be configured for a wide range of engines. This observer device, using the logic developed in the simulator, could be an extremely compact unit. It could provide improved control of fuel, speed, temperature, and pressure dynamics. Such a circuit could be incorporated in one



or two monolithic LSI chips, to be externally trimmed for a particular engine or operating condition.



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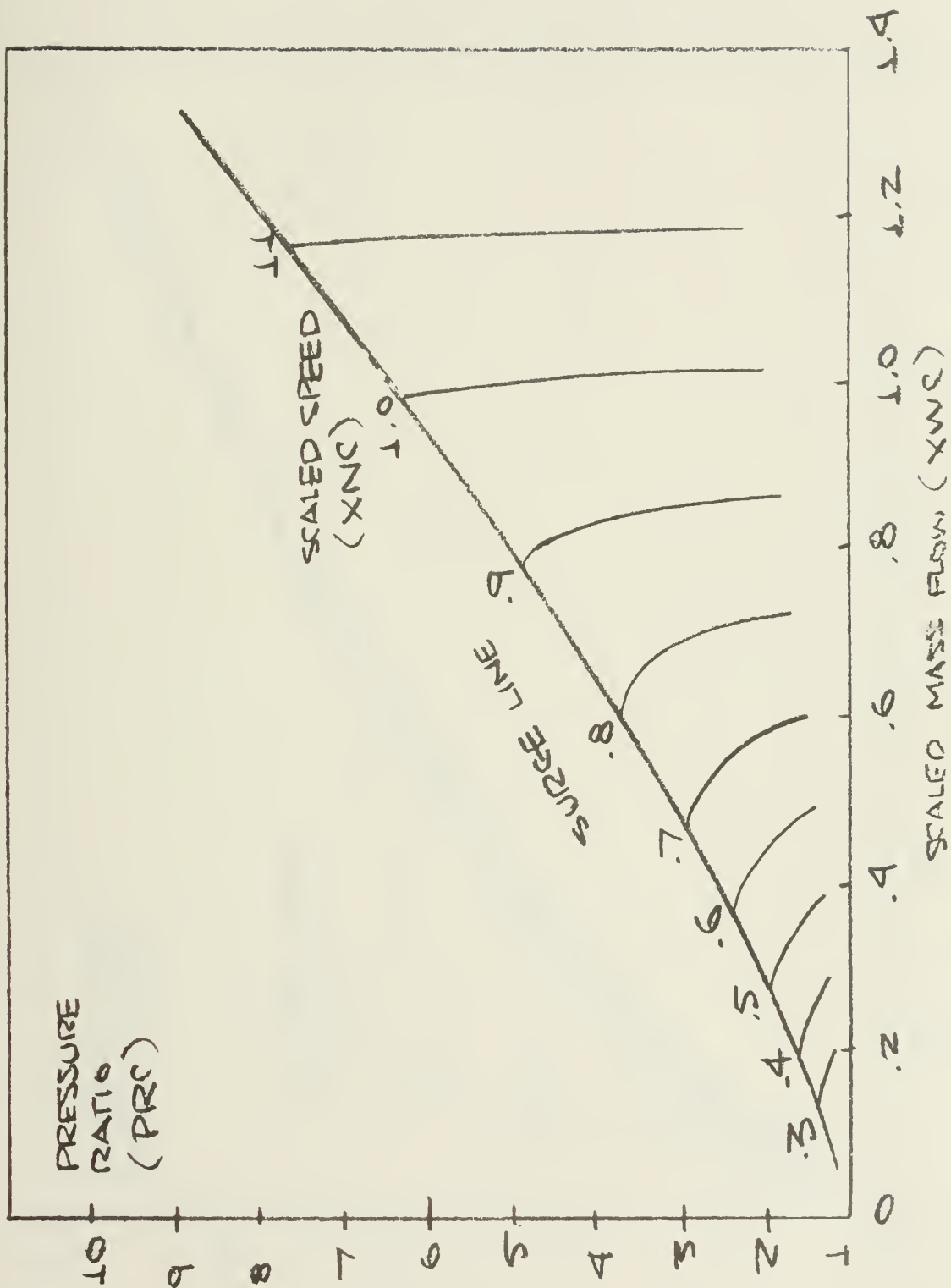


APPENDIX I

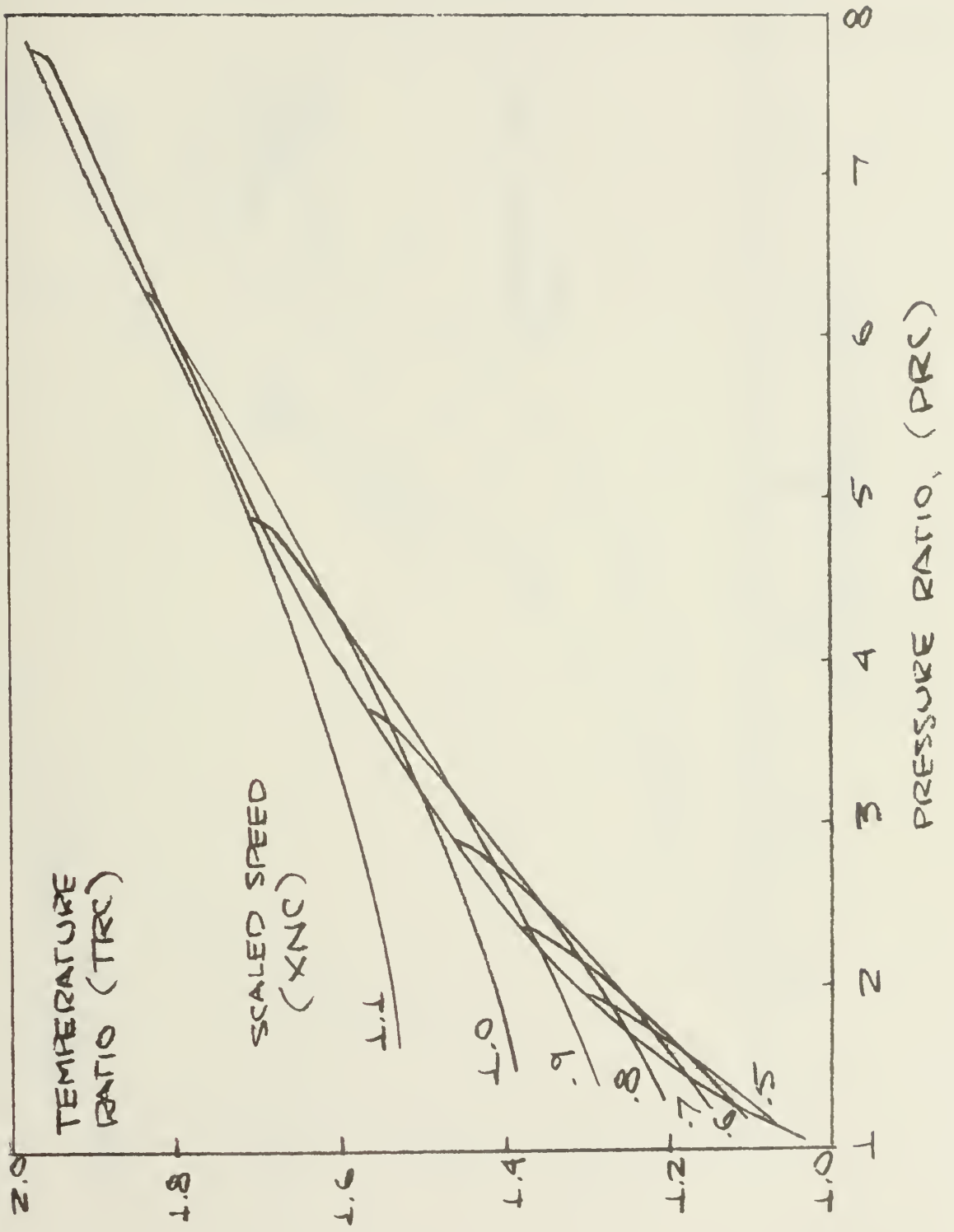
MACHINE CHARACTERISTICS



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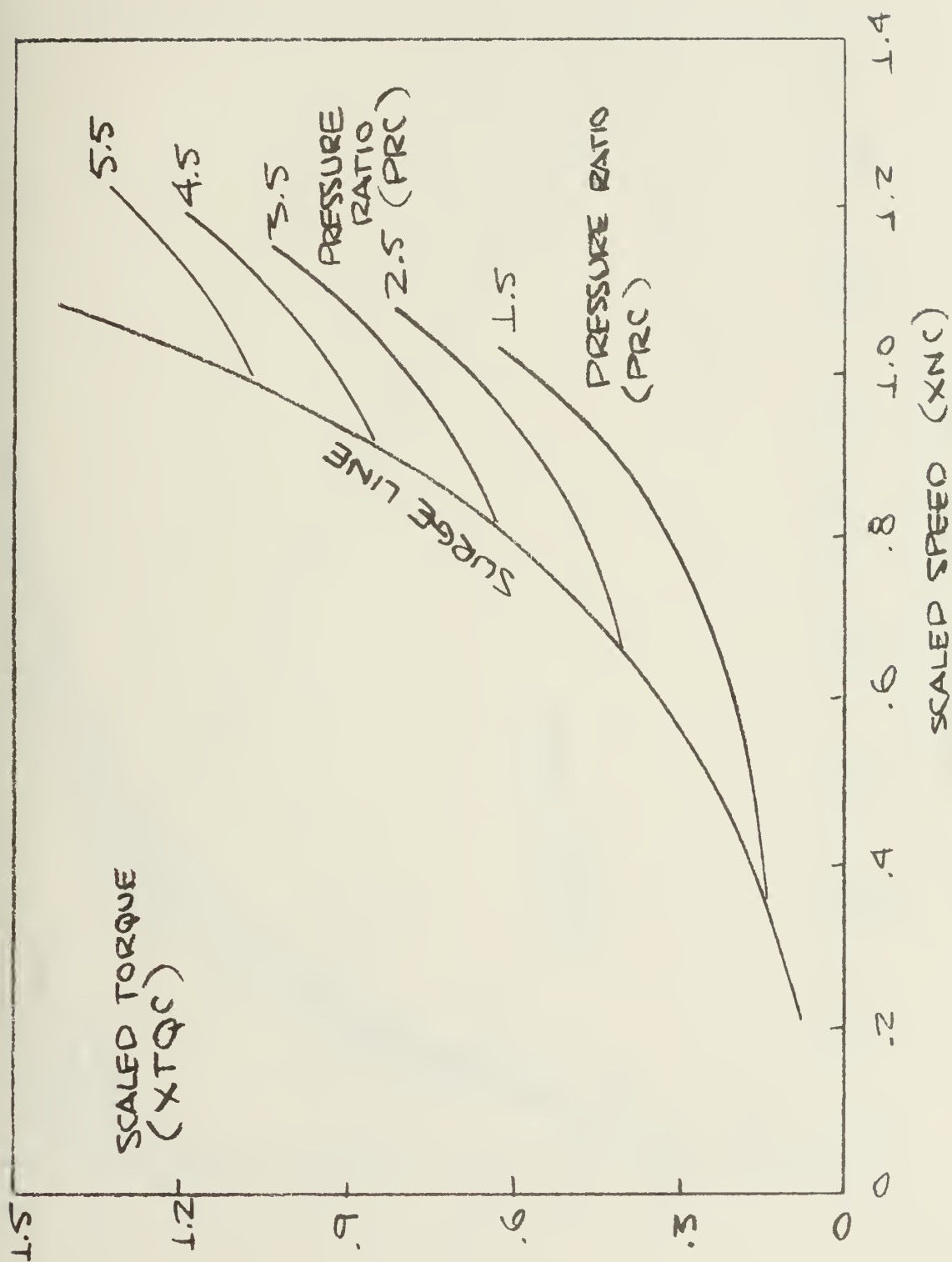




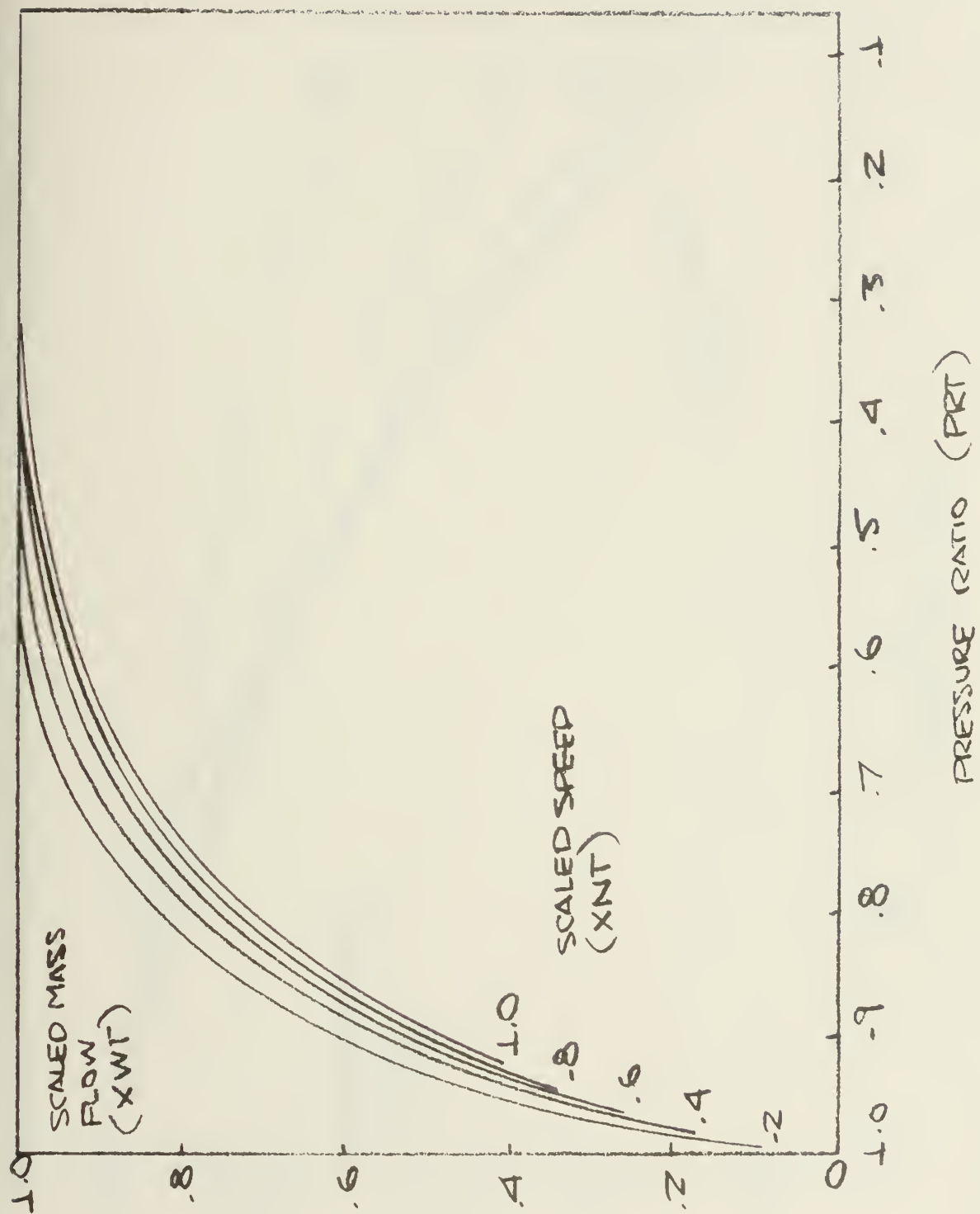




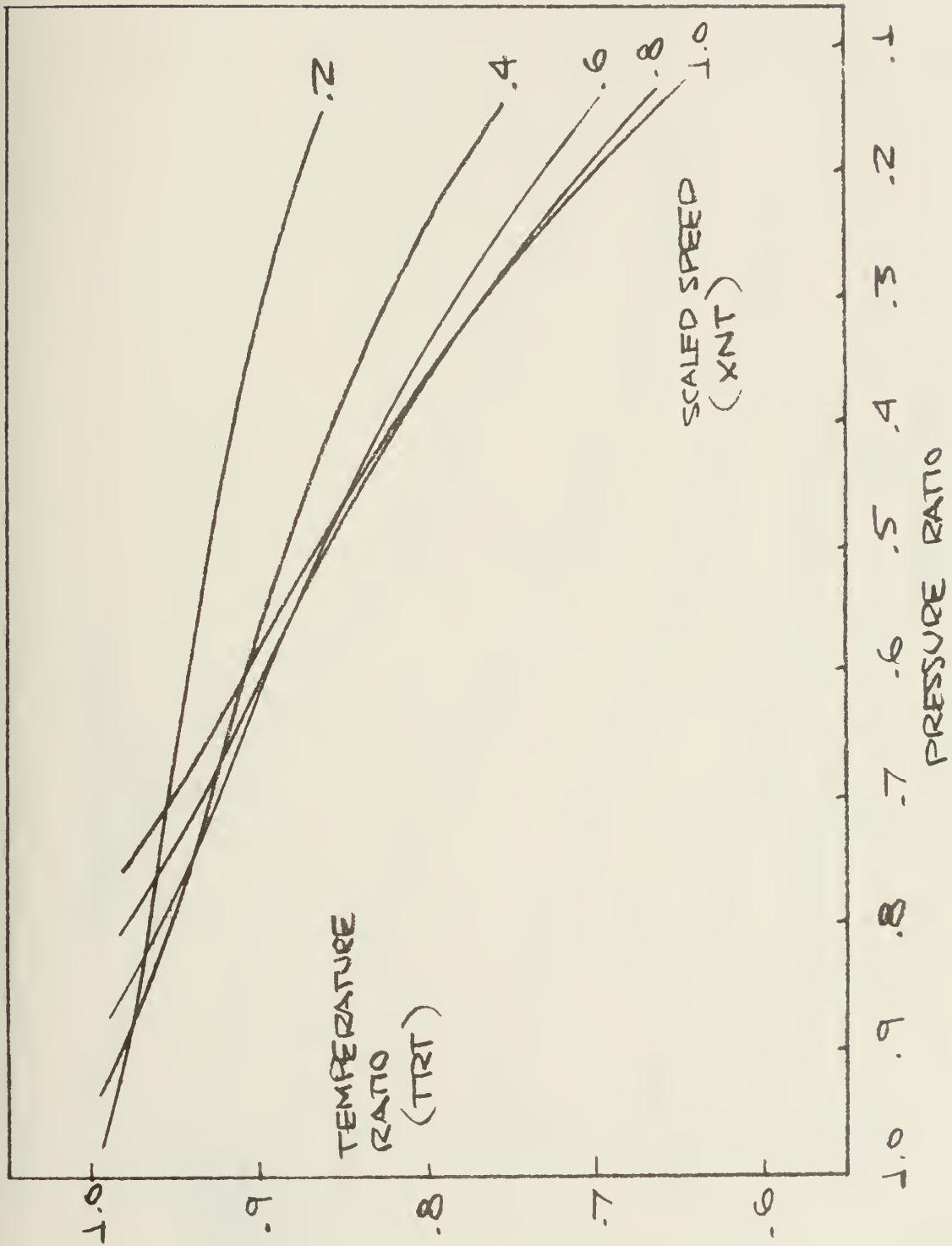




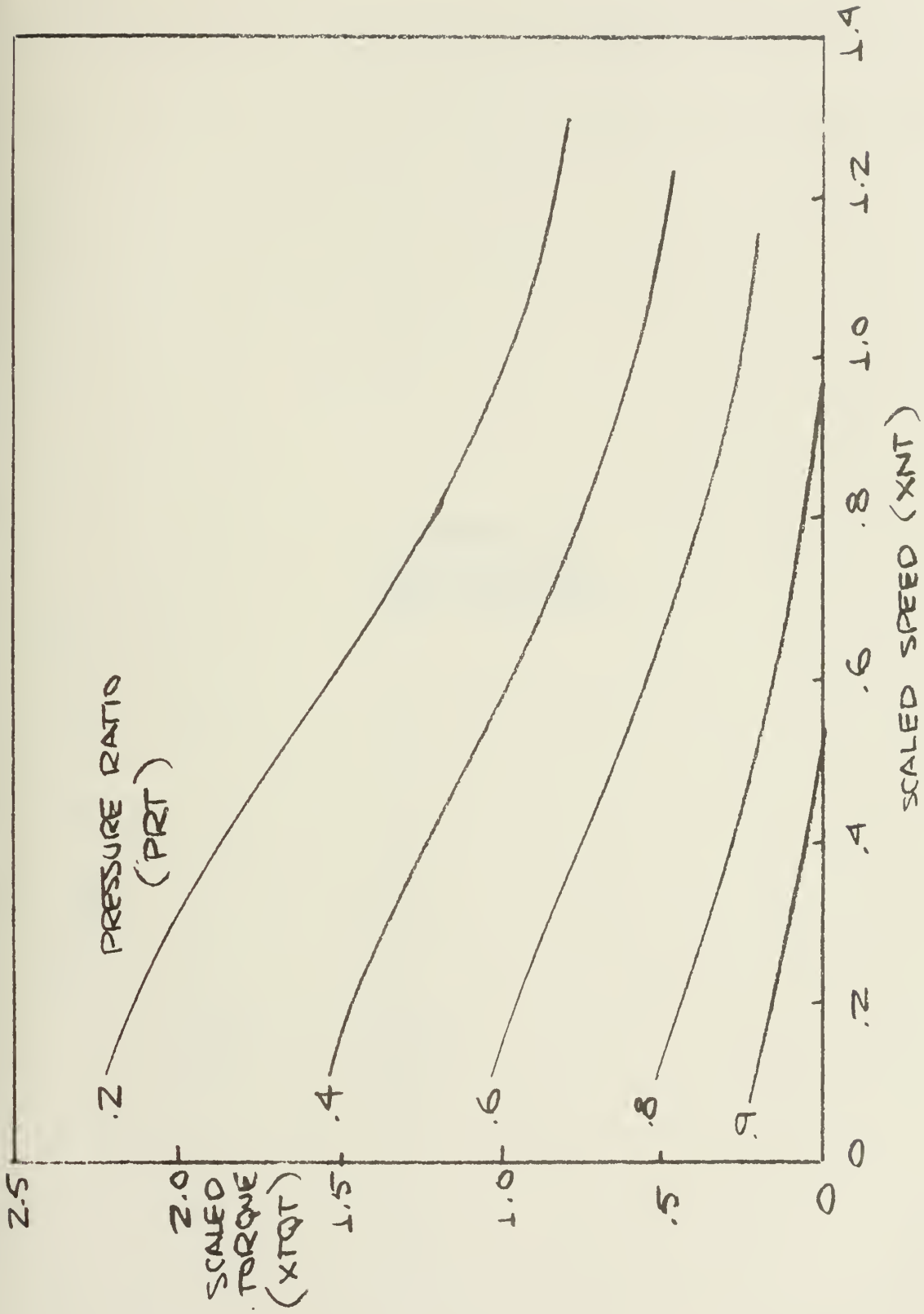
















APPENDIX II  
STATE EQUATIONS



# STATE EQUATIONS

I.

$$\dot{P}_{O3} = \left( \frac{nRT_{O4}}{V_{OL}} \right) [\dot{m}_3 - \dot{m}_4]$$

$$\dot{P}_{O3} = \left( \frac{nRT_{O4}}{V_{OL}} \right) [(x_{MC})(\dot{A}_C)(x_{WC}) - (x_{MT})(\dot{A}_T)(x_{WT})]$$



II.

$$\dot{T}_{04} = \left( \frac{n-1}{n} \right) \frac{R T_{04}}{P_{03} V_{04}} \left[ \dot{m}_3 T_{03} + \dot{m}_F \left( \frac{h_{LHV}}{C_P} \right) - \dot{m}_4 T_{04} \right]$$

$$\dot{T}_{04} = \left( \frac{n-1}{n} \right) \frac{R T_{04}}{P_{03} V_{04}} \left[ (x_{MC}) (A_C) \left( 1 - \left( \frac{P_{02}}{P_{03}} \right)^{\frac{\gamma-1}{\gamma}} \right) \right] \left[ T_{02} + T_{02} \left( \frac{P_{02}}{P_{03}} \right)^{\frac{\gamma-1}{\gamma}} \right] +$$

$$\dot{m}_F \left( \frac{h_{LHV}}{C_P} \right) - T_{04} (x_{MT}) (A_T) (x_{WT})$$

$$\text{WHERE } x_{MC} = \frac{P_{02}}{\sqrt{T_{02}}} \left( \frac{\dot{m}_3 \sqrt{T_{02}}}{P_{02}} \right)_{\text{DESIGN}}$$

$$x_{MT} = \frac{P_{03}}{\sqrt{T_{04}}} \left( \frac{\dot{m}_4 \sqrt{T_{04}}}{P_{03}} \right)_{\text{DESIGN}}$$



III.

$$\dot{N}_G = \frac{30}{\pi I} \left[ \tau_2 - \tau_1 - \tau_3 \right]$$

$$\dot{N}_G = \frac{30}{\pi I} \left[ \frac{30 \rho T_{01}}{\pi} \left[ \frac{N_G}{N_G} \right] \left( x_{MT} \right) (AT) (1 - PRT)^{BT} \right] (2 - (1 - (1 - \sqrt{1 - .8(x_{MT})^{1.25}})^2)) \left[ (1 - (PRT)^{GET}) \right] - (x_{MC})(AC) \right] - \tau_3 \left[ (x_{WC})(T_{02}) \left( \frac{T_{RC} - 1}{N_G} \right) - \tau_3 \right]$$



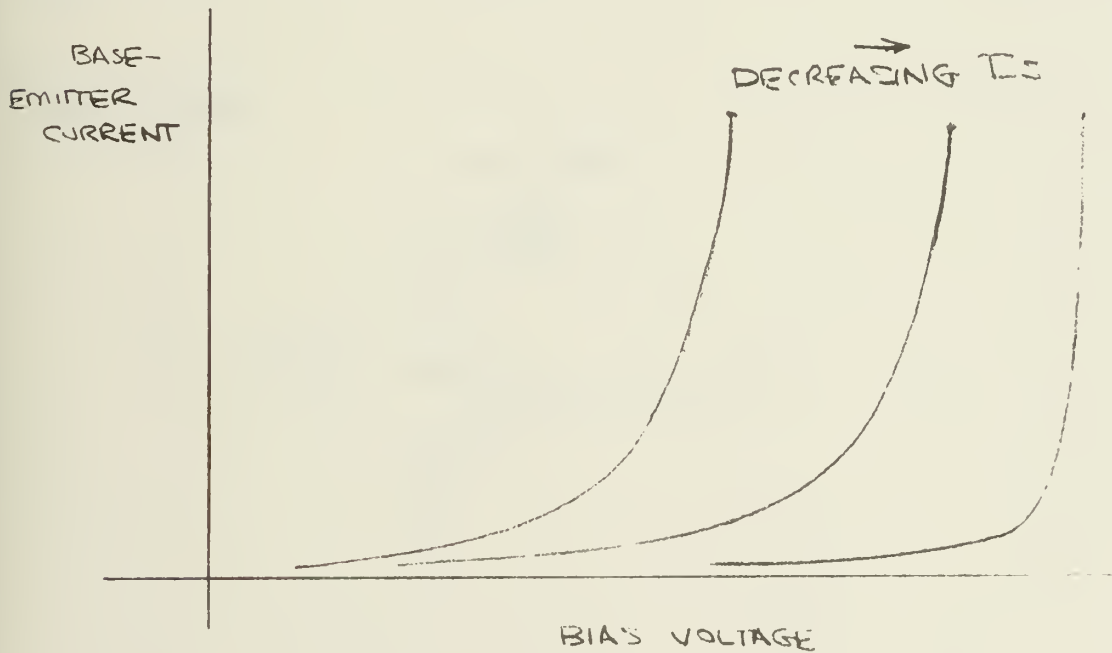


APPENDIX III

Test Data

A. TEST 1 AND 2

From reference data,<sup>19</sup> the transdiode voltage current characteristics was estimated for various values of saturation current. The log circuits diagrammed subsequently for the 2N2222 and 2N2904 transistors demonstrated low base-emitter saturation currents, thus making the exponential characteristic curve more abrupt.



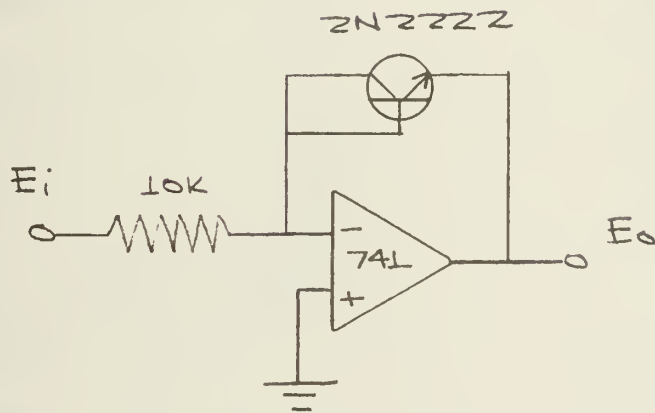


Test Data

$$I = I_s e^{(E_o/26\text{Mv})}$$

<u>E<sub>O</sub> (Mv)</u>	<u>I (MA)</u>	<u>Estimated I<sub>S</sub> (PA)</u>
470	.004	5.04 x 10 <sup>-11</sup>
548	.015	1.05 x 10 <sup>-11</sup>
581	.045	8.87 x 10 <sup>-12</sup>
609	.121	8.13 x 10 <sup>-12</sup>
616	.178	9.14 x 10 <sup>-12</sup>

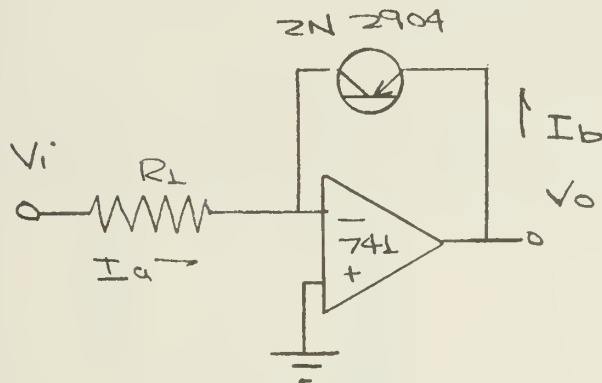
Test Circuit





TEST 2

CIRCUIT DIAGRAMS AND DATA



$R_L = 10K$

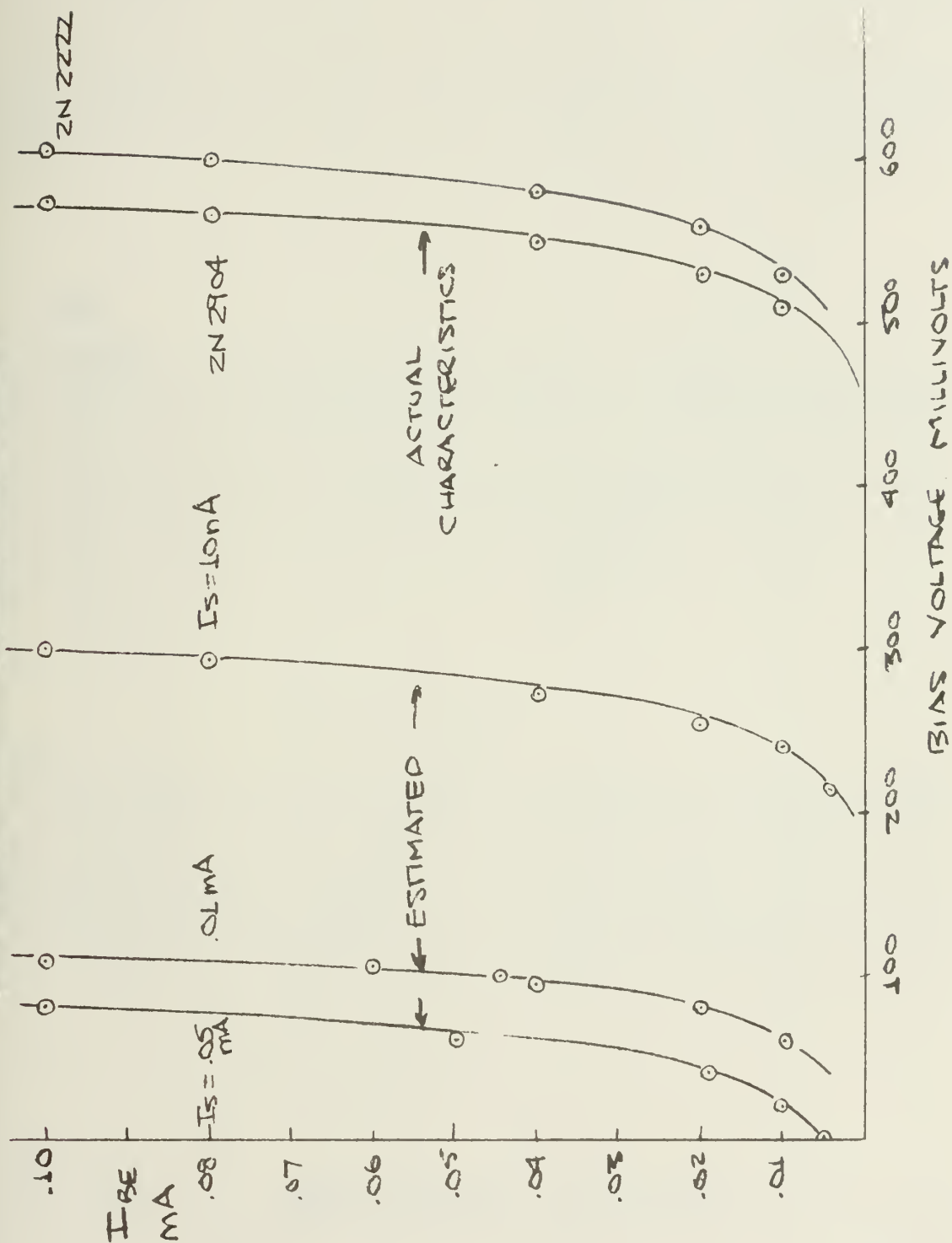
<u><math>V_i</math> (VOLTS)</u>	<u><math>I_a</math> (mA)</u>	<u><math>V_o</math> (mV)</u>	<u><math>I_b</math> (mA)</u>
-4.82	.161	580	.147
-4.32		580	.137
-3.95	.133	580	.133
-2.12	.070	565	.074
-1.71	.057	559	.060

$R_L$  SWITCHED TO 30K

- .003	384
- .011	423
- .021	445
- .044	465
- .071	476
- .106	487
- .142	502
- .175	509
- .251	511
- .430	523

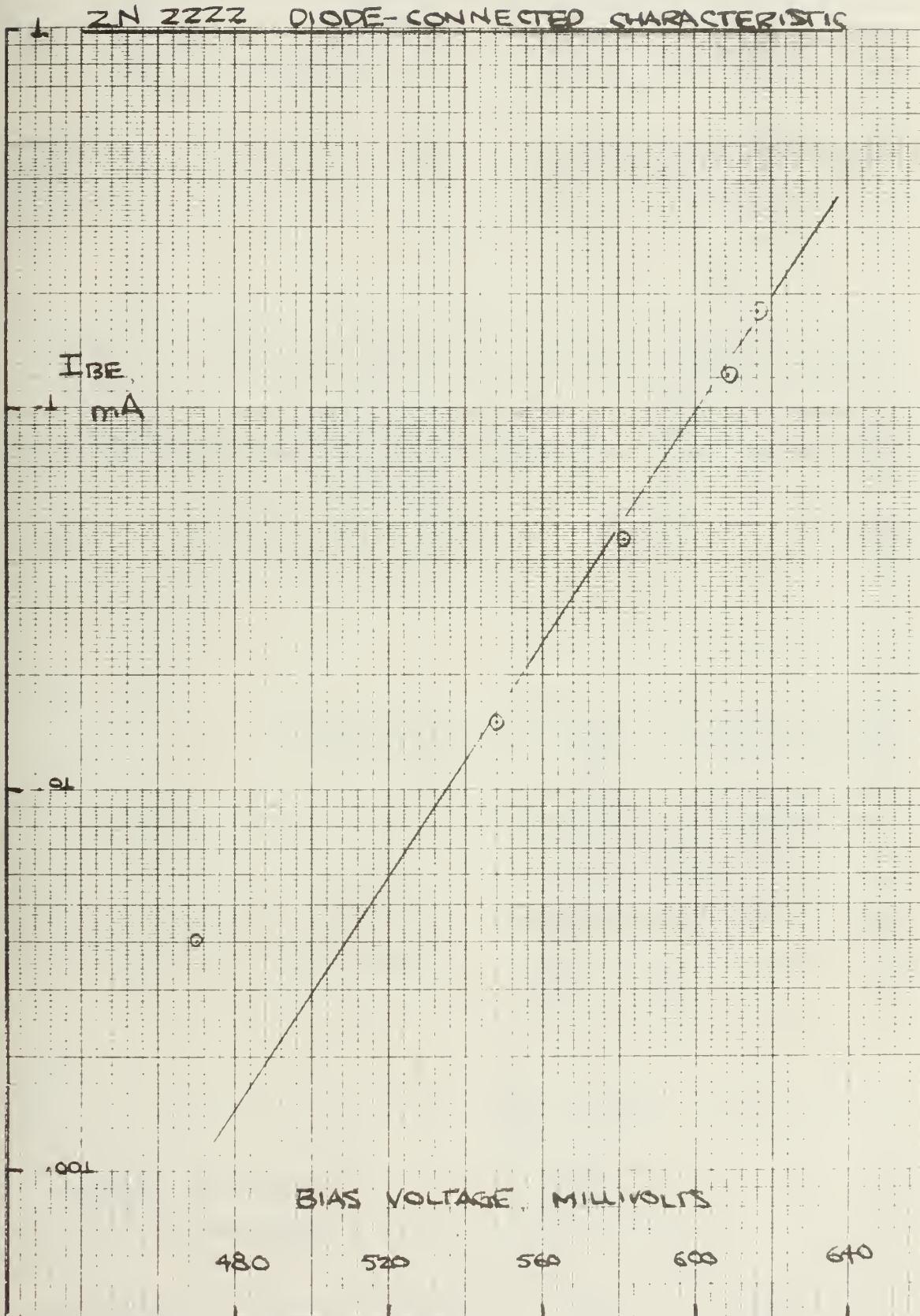


# TRANSISTOR CHARACTERISTICS

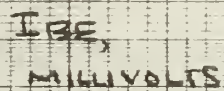












BIAS VOLTAGE  
MILLIVOLTS



38

400

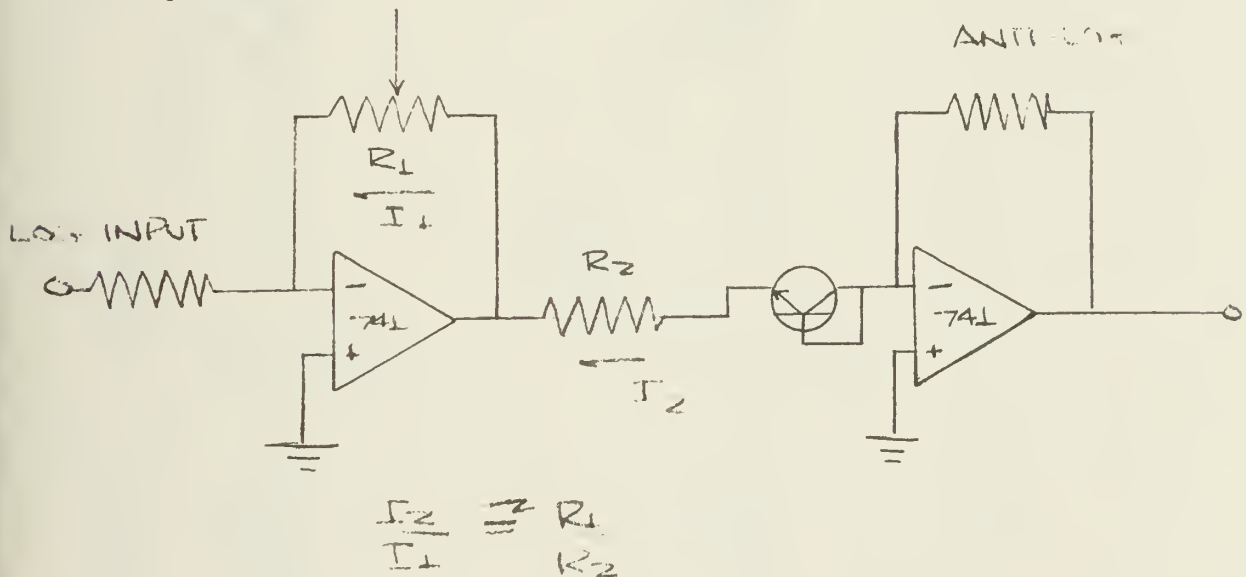
55

8



B. TESTS 3 - 6

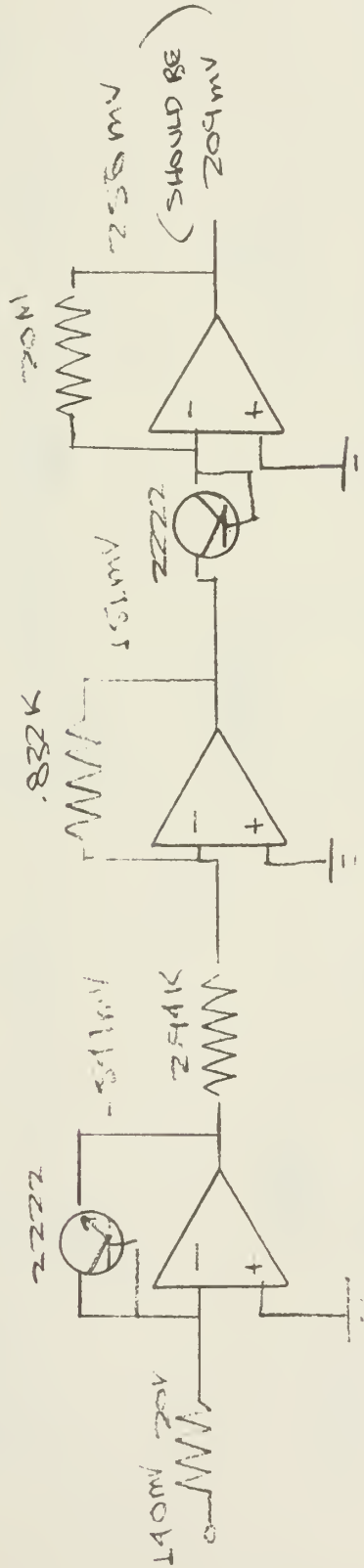
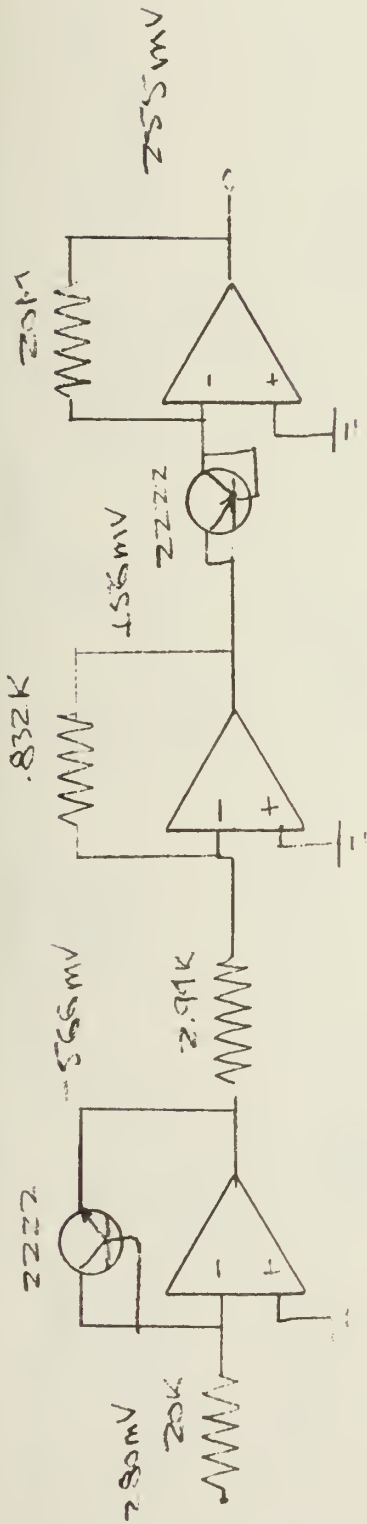
The fixed exponentiation of a variable was evaluated in Tests 3 - 6. Tests 3, 4, and 5 were failures as the essential concept of current division to achieve the exponentiation was not understood. Test 6 demonstrates the use of a resistive network which accomplished the current manipulation. It is now apparent that as a result of the characteristics of the simple log configuration, current division can be easily accomplished by assigning the appropriate resistor value to the antilog input. This is diagrammed:





TEST 3

CIRCUIT DIAGRAMS AND DATA



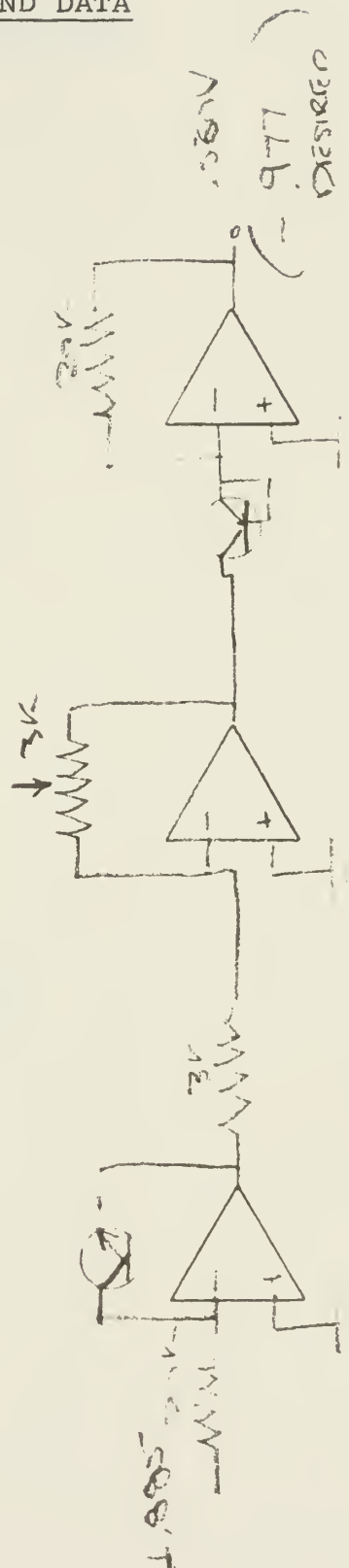
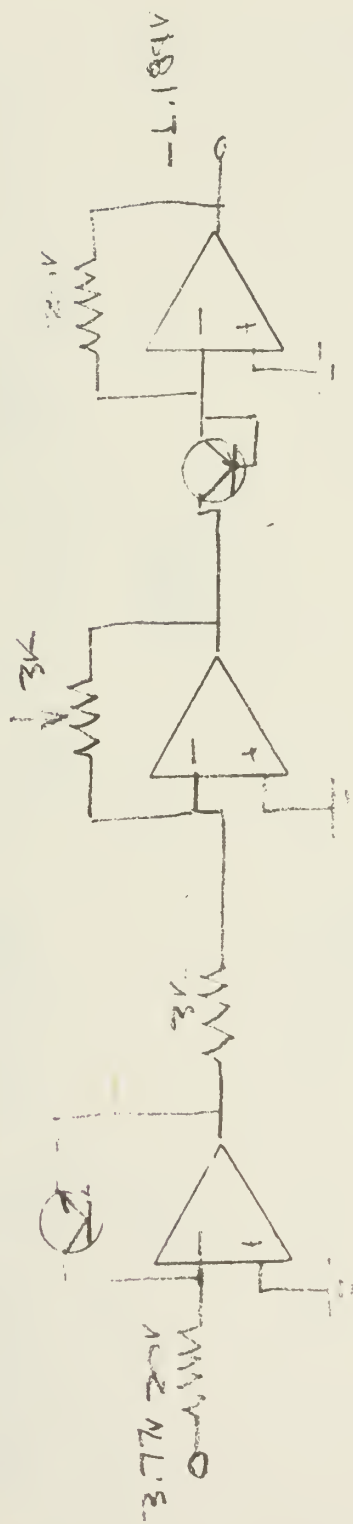
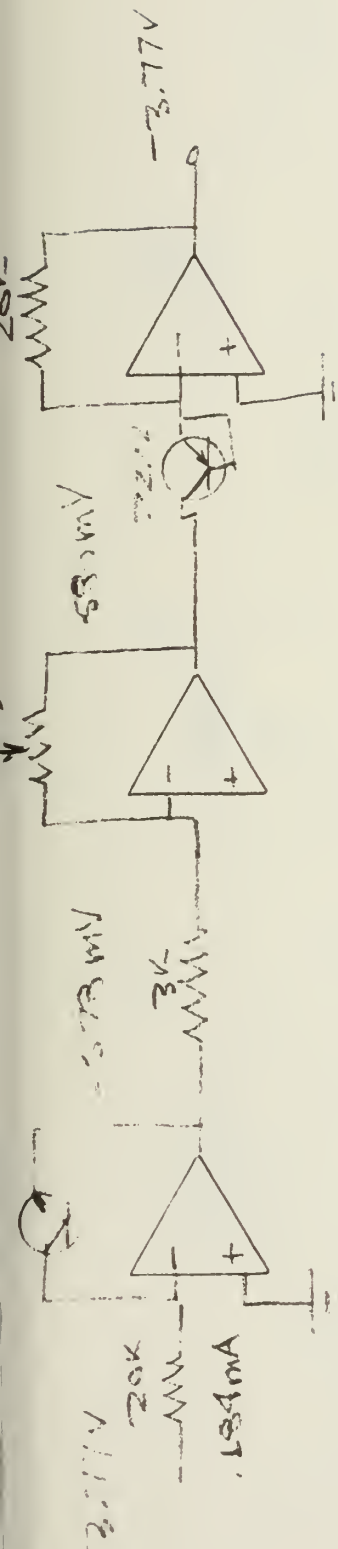
DESIRED FUNCTION:  $Y = X$   
 $\uparrow$   
 $10 < K < 10$





TEST 4

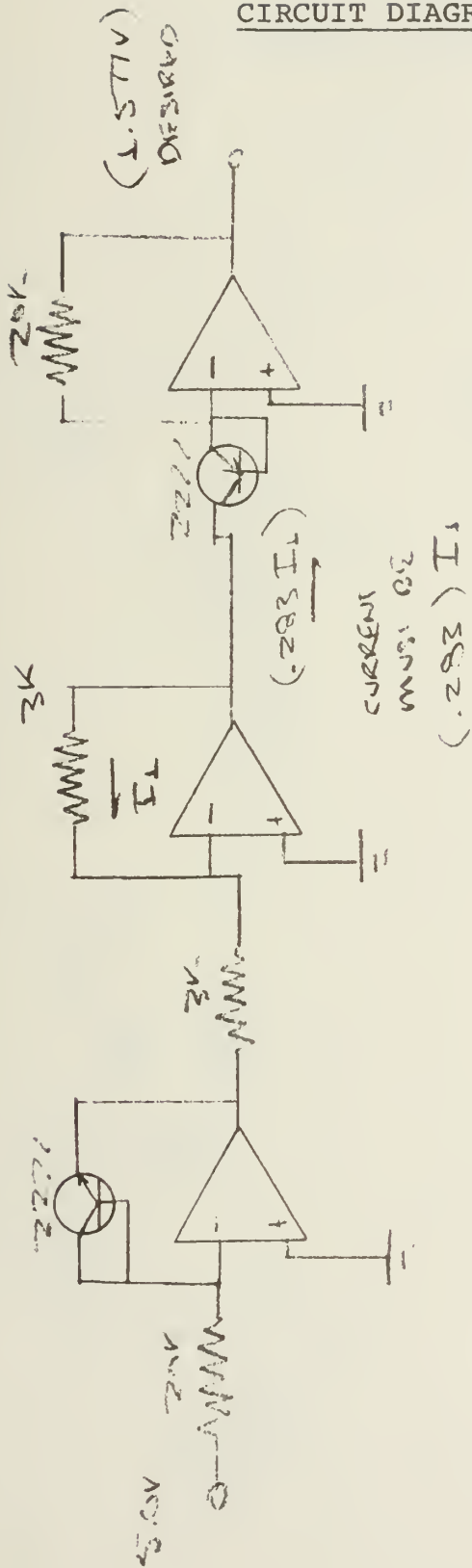
CIRCUIT DIAGRAMS AND DATA





TEST 5

CIRCUIT DIAGRAMS AND DATA

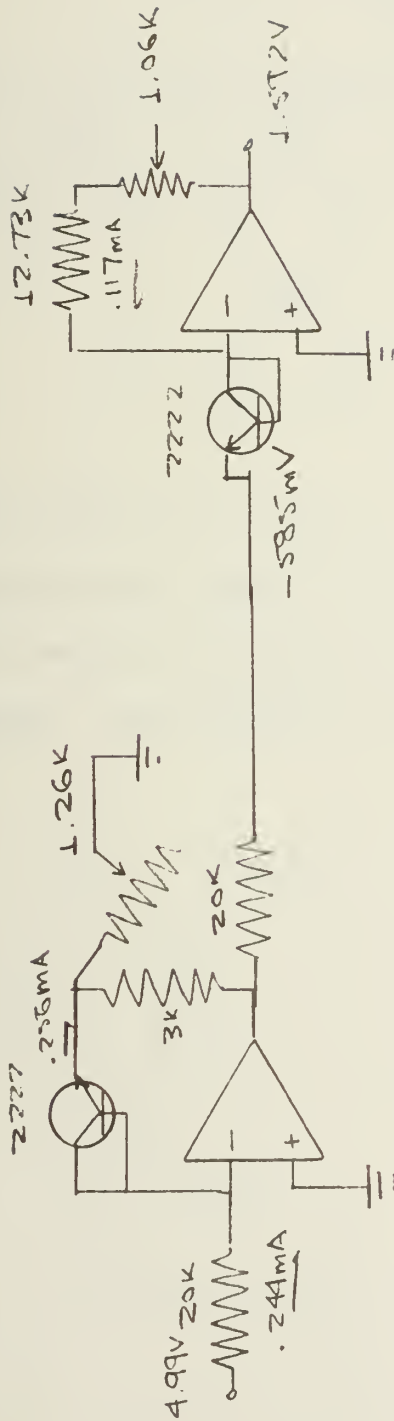


NO. 1000 0.5



TEST 6

CIRCUIT DIAGRAMS AND DATA



FUNCTION DESIRED:  $Y = X \cdot 283$   
 $1.57 \pm 5$

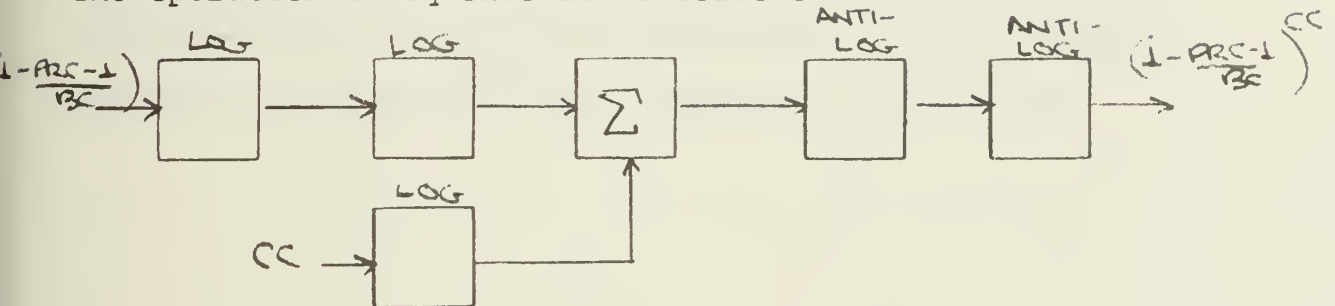


C. TEST 7

It is desired to validate the circuit segment for the operation

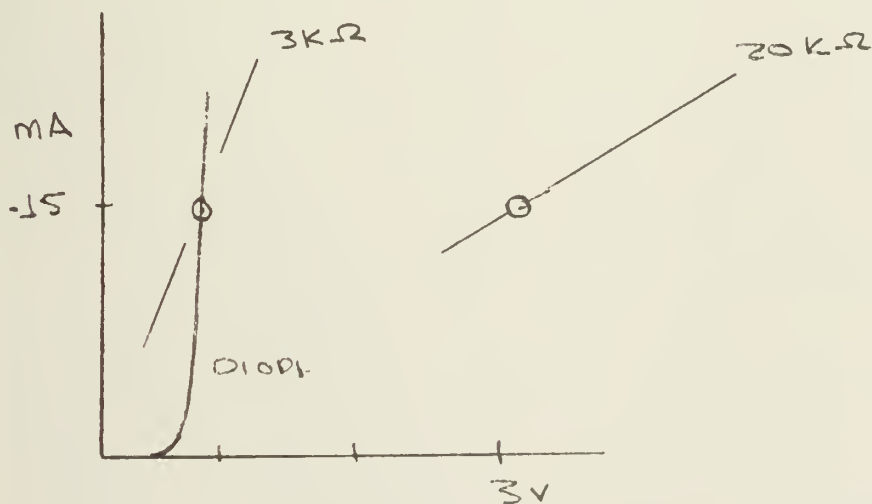
$$\left(1 - \left(\frac{PRC-1}{BC}\right)\right)^{CC}$$

The operational sequence is as follows:



The circuit described in the test data was set up to evaluate the feasibility of the operation.

The desired operating characteristic of the cascaded logger - antilogger is pictured as follows:







The first log generator establishes a voltage - current relation based on the diode and 20K resistor. Consider the second log generator to be superimposed in the first with a 3K resistor and a second diode.

The cascaded antilog - configured amplifiers displayed gross thermal sensitivity. The simple OP amp - transdiode configuration is unsuitable for this cascaded operation without additional temperature compensation.







D. TEST 8

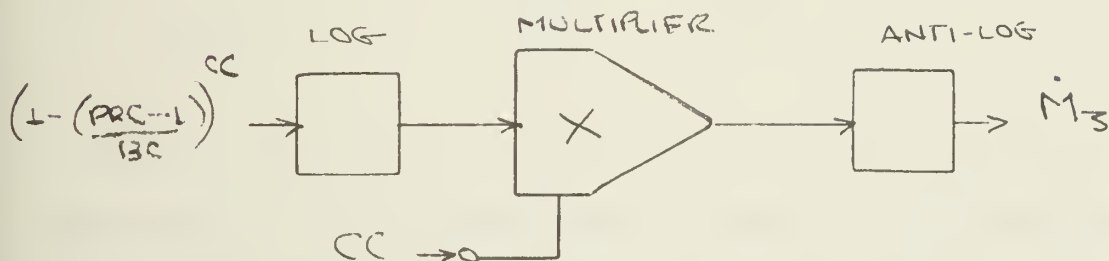
i. Purpose

Test 8 was conducted to verify the variable exponent part of the analog circuitry, using the four quadrant multiplier.

The equation of interest is written below:

$$\dot{M}_3 = \left( 1 - \left( \frac{PRC-1}{BC} \right)^{CC} \right)$$

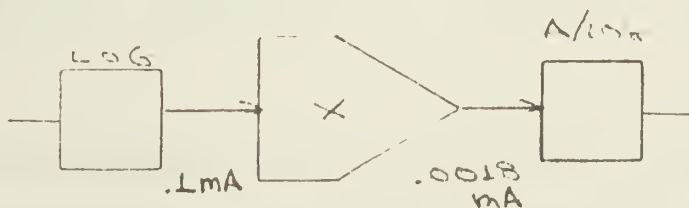
LOOKING AT THE LOGIC FLOW:



ii. Methodology

The functional method used to exponentiate to a variable power was to regulate the multiplier output current in the ratio called for by the exponentiation. For example:

$$IF \quad CC = .018$$

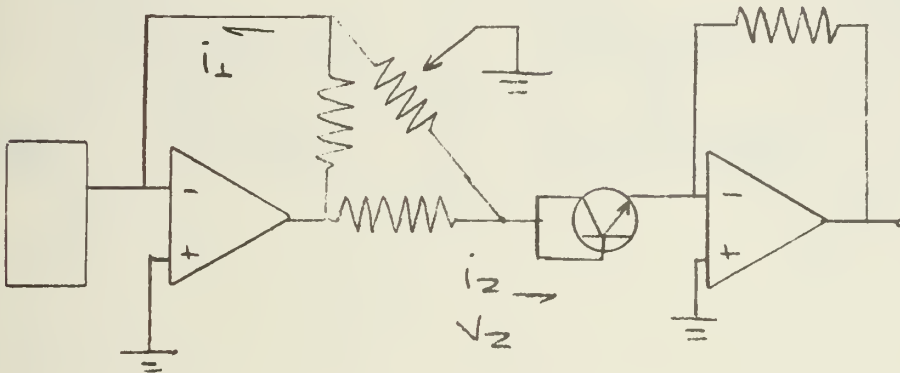




For multiplier scaling the integral buffer OP amp in the 2208 was used for the current division with a resistive bridge as follows,

MULTIPLIER + BUFFER OA

ANTILOG



such that  $i_1 = i_2$  and the output voltage of the multiplier is thus controlling the current input to the antilog amplifier transdiode.

The multiplier output is nominally:

$$V_z = \frac{V_x V_y}{10}$$

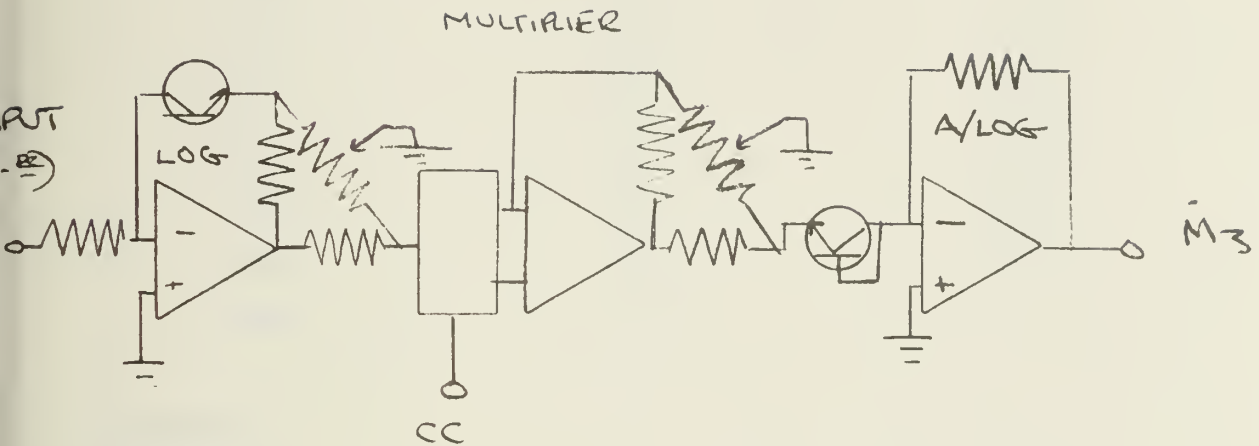
TO OBTAIN  $V_z \leq 600 \text{ mV}$ , RESISTANCE VALUES MUST BE SELECTED TO GET THE DESIRED RANGE OF ANTILOG INPUTS.

It was determined that a resistive bridge must be used for the multiplier input from logging OA, sketched





as follows:



so by modulation of CC input, the desired current ratios across multiplier can be achieved.

### iii. Data

A 5 minute observation of the steady state output voltage was made to investigate the circuit thermal stability.



10 Second Samples

-Volts x 10

.630	.650	.639
.635	.643	.634
.625	.671	.634
.667	.649	.640
.644	.651	.642
.633	.651	.629
.630	.651	.631
.624	.641	.626
.661	.652	

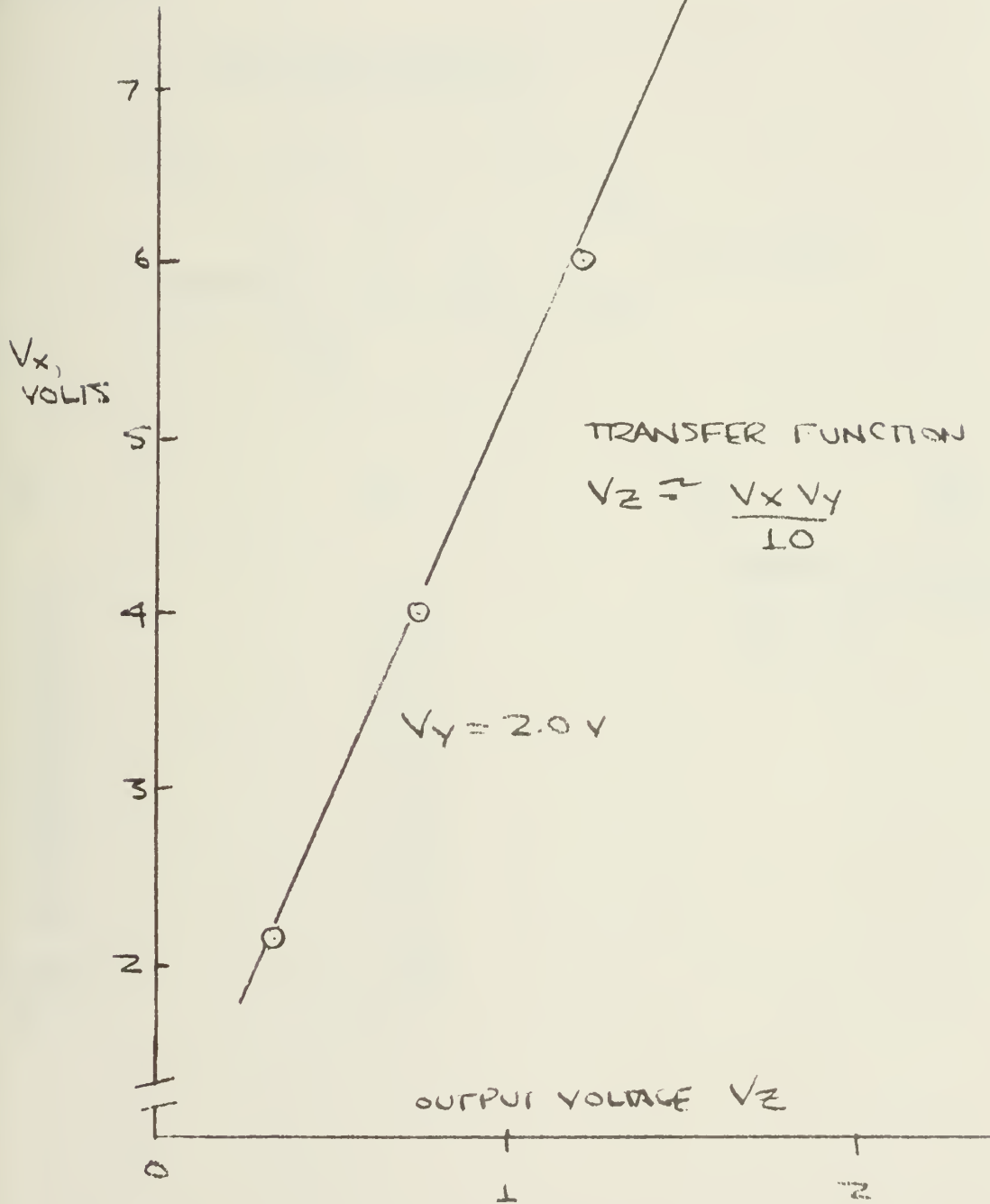
26 Samples

MEAN = .642

STD DEVIATION = .0137



EXAR 2208 MULTIPLIER  
CHARACTERISTIC



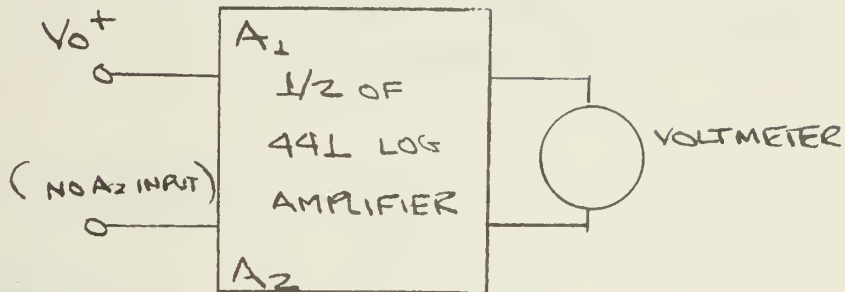


E. TEST 9

i. Purpose

To investigate the Texas Instrument TL 441 log amplifier.

ii. Log Configuration



<u><math>V_o</math></u>	<u><math>V_{y\bar{y}}</math></u>	<u><math>V_o</math></u>	<u><math>V_{y\bar{y}}</math></u>
.002	.023	(VIRTUALLY IDENTICAL FOR INPUT AT PIN A2)	
.010	.060		
.020	.097		
.030	.143		
.080	.251		
.100	.275		
.150	.313		
.200	.343		
.300	.379		
.500	.416		
1.003	.423		
2.00	.427		

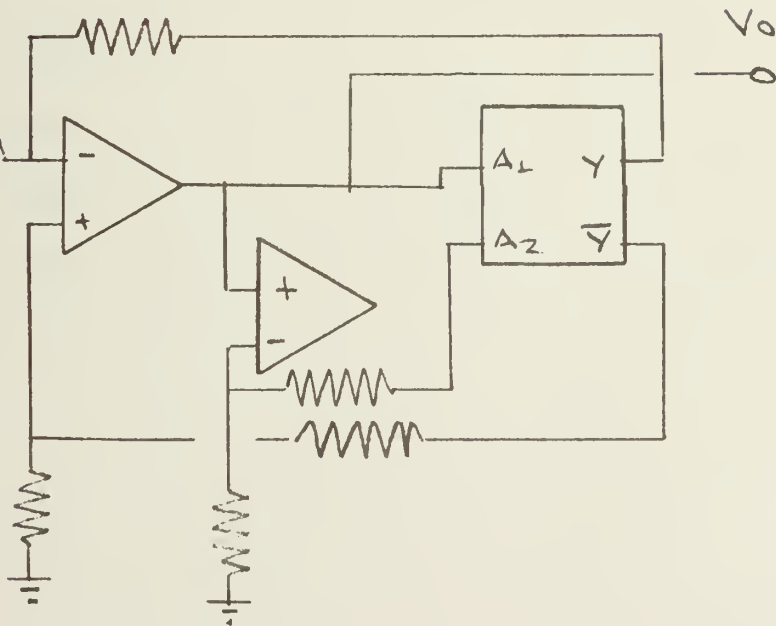
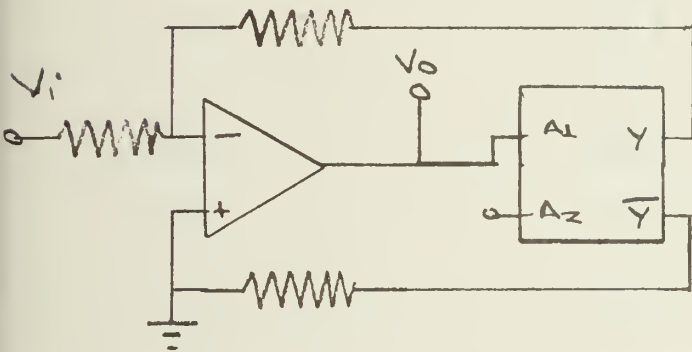




The output characteristic  $V_{Y\bar{Y}} = \sum \log A_1 + \log A_2$  verified. From the plot, "TL 441 transfer characteristics, the log amplifier has 1 1/2 decades of dynamic input range, from 20 - 600 mV.

### iii. Antilog Configuration

TWO ALTERNATE WIRINGS WERE INVESTIGATED:





Antilog Configuration 1

<u>V<sub>i</sub></u>	<u>V<sub>o</sub></u>	<u>V<sub>o</sub> + .028</u>
-.001	-.028	0
-.010	-.026	.002
-.049	-.018	.013
-.100	-.002	.050
-.200	.025	.053
-.300	.063	.091
-.400	.171	.199
-.500	.296	(.296)
-.600	.296	SATURATION
-.701	.296	
-.120		
-.150	(NO DATA)	
-.180		
-.350	.098	.126
-.450	.293	(.293)
		SATURATION
-.420	.213	.241



The antilog data is graphed in "TL 441 antilog characteristics" for configuration 1. Configuration 2 will expand the antilog dynamic range somewhat above the 1 1/2 decades available for configuration 1.



# TL 441 LOG CHARACTERISTICS

.5

DIFFERENTIAL  
OUTPUT VOLTAGE,  
VOLTS

$\times 7$

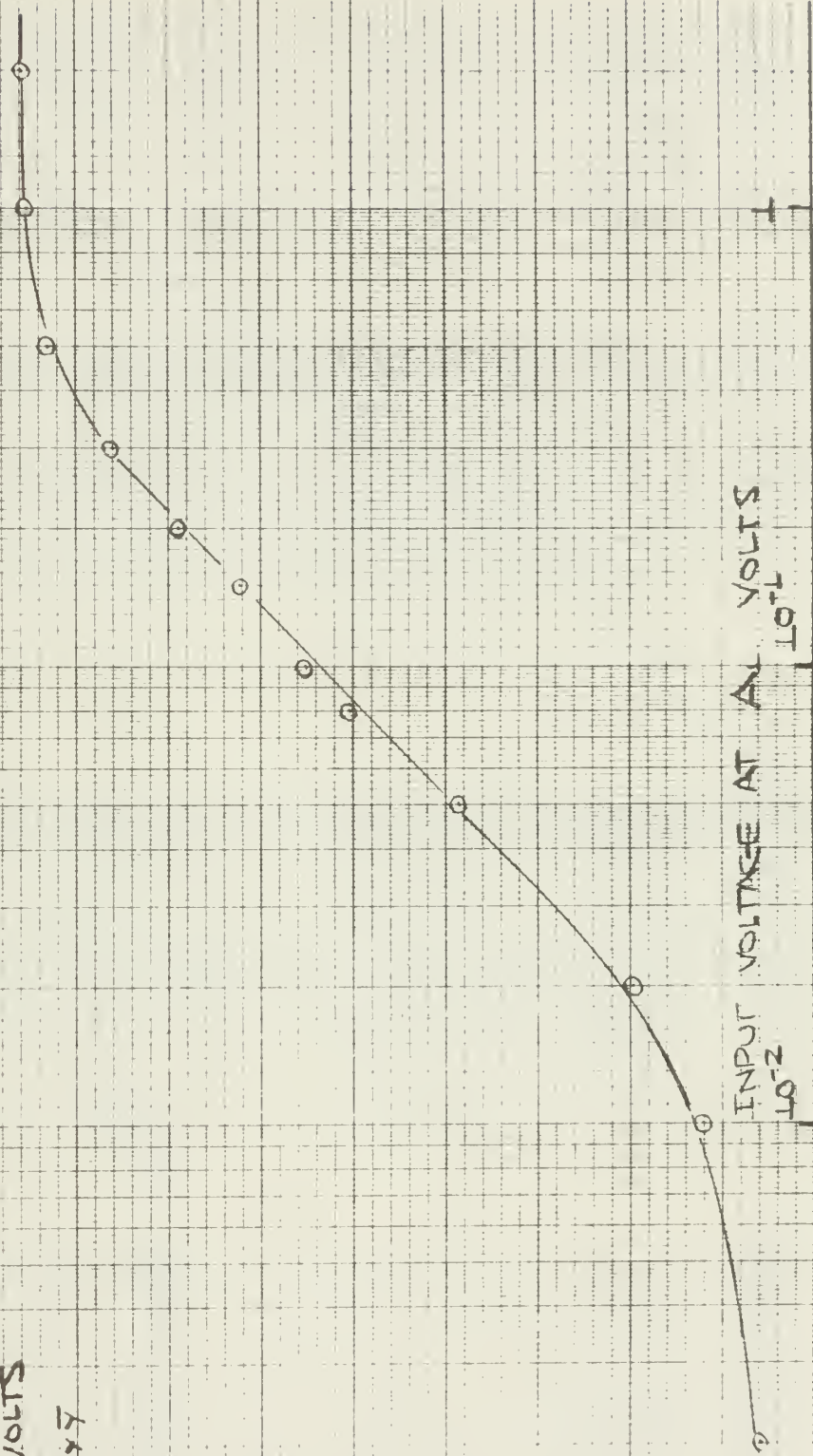
3

2

1

INPUT VOLTAGE AT  
 $10^{-2}$  VOLTS

1







TL 441 ANTILUX CHARACTERISTICS

-5

-4

INPUT VOLTAGE, VOLTS

-3

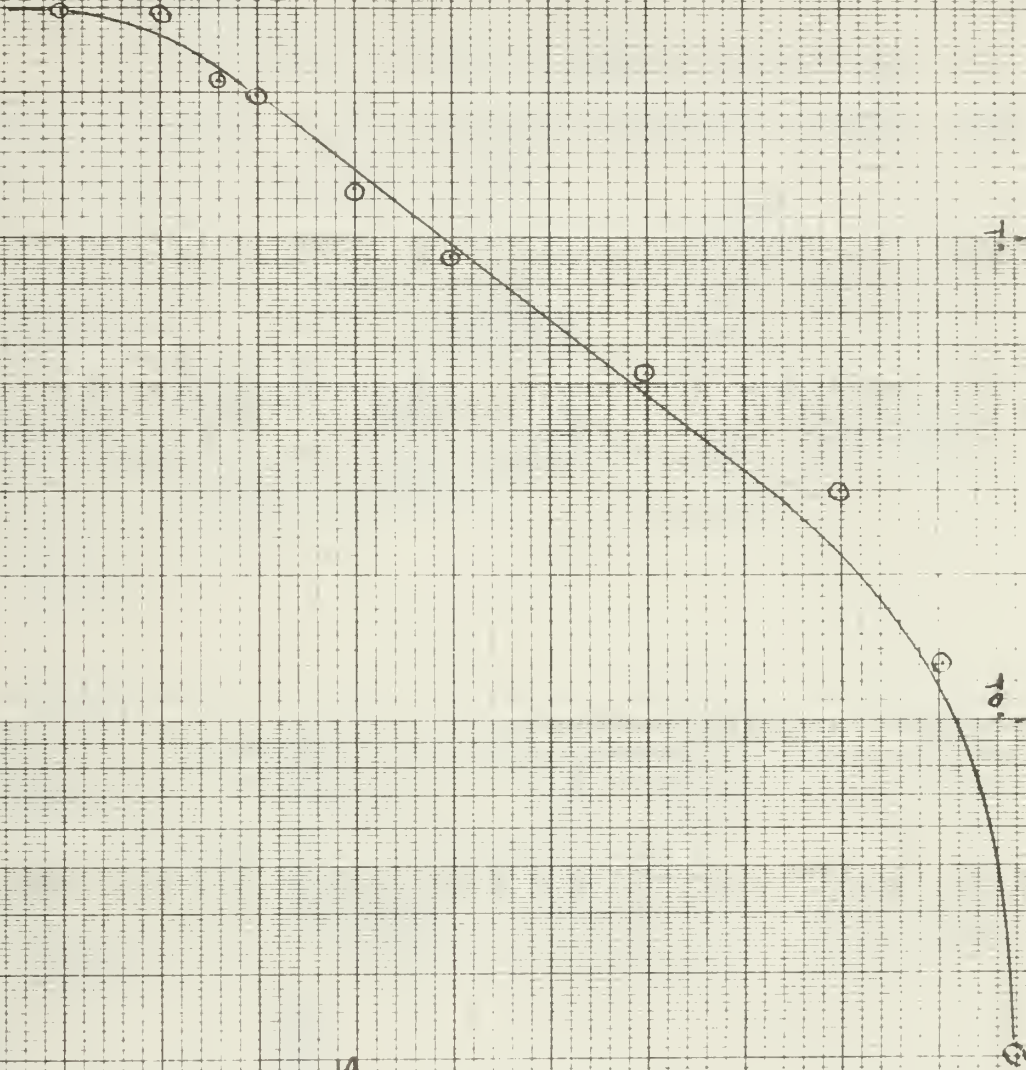
-2

-1

OUTPUT VOLTAGE,  
VOLTS .90L

.01

1

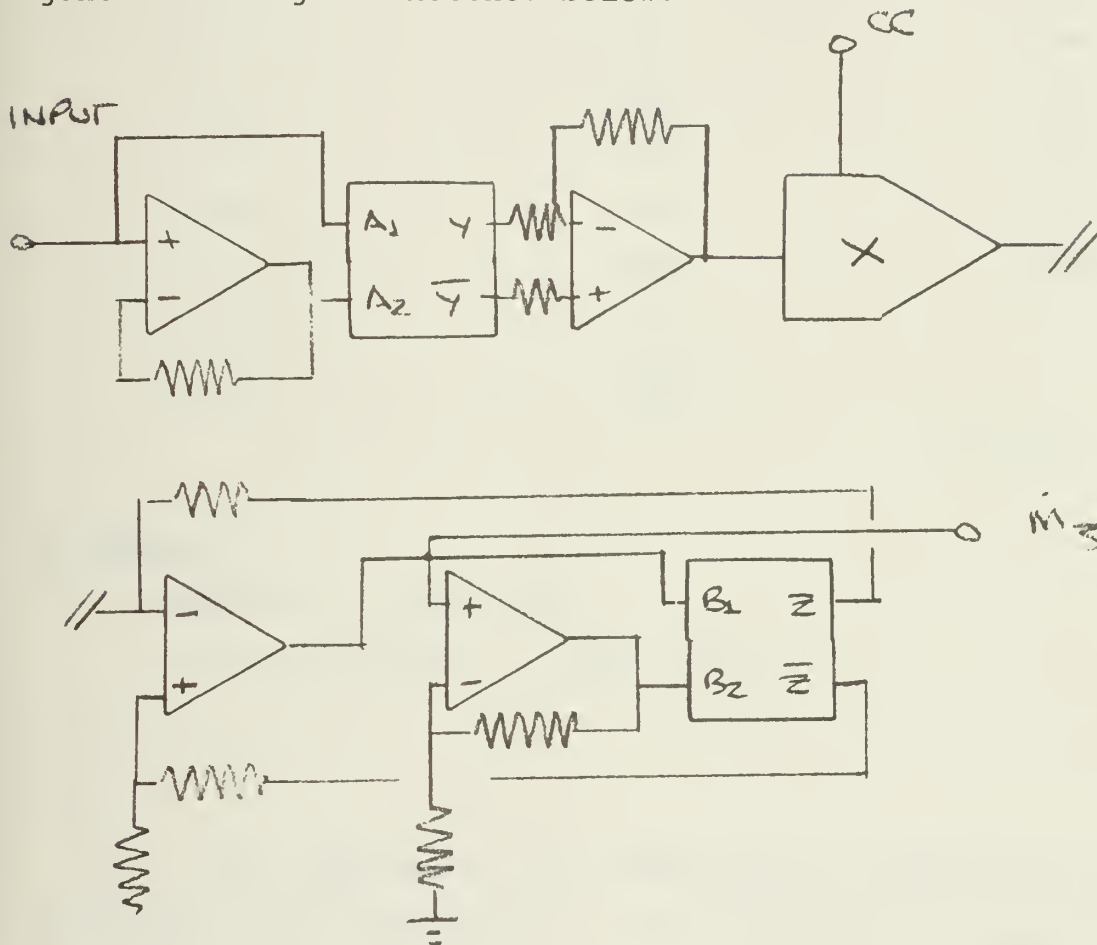




F. TEST 10

i. Purpose

To incorporate the 441 log amplifier and the quadrant multiplier in the variable exponent circuit. The general wiring is sketched below:



ii. Steady State Performance

The circuit demonstrated the ability to track a desired variable exponent. It was of primary interest to determine if the thermal properties were improved over the simple log-fed multiplier circuit.



The following data was generated:

Variable Exponent Test

1.50	1.39	1.45
1.57	1.47	1.37
1.55	1.47	1.46
1.47	1.42	1.41
1.55	1.47	1.38
1.52	1.39	1.41
1.45	1.45	1.48
1.52	1.41	
1.50	1.46	

(VOLTS)

25 Samples

MEAN = 1.474 VOLTS

STD DEVIATION = .052 VOLTS

This represents a 50% reduction in the drift over the simple log configuration. Since the 441 log amplifier is fairly cheap, it is substituted for the simple log configuration.



APPENDIX IV

DYSYS SIMULATION





```

SUBROUTINE POSI*
COMMON T,DT, Y(20), F(20), STIMT,FTIME,NEWDT,IEWPT,N
REAL NG
IF(NEWDT.NE.-1) GO TO 1
GFC = .2431
GET = .2536
1 IF(NEWDT.EQ.0) GO TO 2
P3 = Y(1)
T4 = Y(2)
NG = Y(3)
Y(5) = P3/16320.
Y(6) = T4/2159.
Y(7) = NG/8000.
PRC = P3/2041.2
PRT = 1./PRC
YNC = NG/7200.
YNT = NG/(SQRT(T4)*172.62)
AC = .75*YNC+.27*YNT**3.0
BC = 3.*YNT**1.5 + 3.*YNT**7.
CC = .01803*(2.-YNT)**5.794
DC = .8906*SQRT(.9-(YNT-.9)*(YNT-.9))
EC = 3.
FC = 1. + YNT+ 3.6*YNT**4.1
XNC = AC*(1.-(PRC-1.)/BC)**CC
EEFC = DC*(1.-((EC-PRC)/(EC-1.))**EC)
TPC = 1. +(PRC**GFC-1.)/EEFC
AT = 1.0006
BT = 3.1 + 1.62/(YNT + .26)
CT = .7
TT = .88*(1.-(1.-YNT)**4.)*.65
ET = 1.-.8*YNT**1.25
YNT = RT*(1.-PRT**BT)**CT
EEET = TT*(1.-(1.-SQRT(1.-ET))/(1.-BT))*(1.-SQRT((1.-BT)/(1.-
1PBT)))
TRT = 1.-EEET*(1.-PRT**GET)
NC = 100.*YNT

```



```

TC = 539.69*TPC
WT = .4936*P3/SCPT(T4)
TE = TU*TRT
TQC = 1799.*XC*(TC-539.)/NG
TOT = 2011.*WT*(TU-TE)/NG
IF(T.LT.ETIME-1.) GO TO 2
PUT POC,PRT,XVC,XNT,AC,PC,CC,DC,EC,XWC,FFFC,TPC,AT,BT,CT,TT,TE,
1XWT,PFET,TBT,AC,TC,WT,TE,TQC,TQT
2 F(1) =890.*(XC-WT)
F(2) = .0684*(WC*TC+70225.*Y(4)-WT*Y(2))
F(3) = .3291*(TOT-TQC-.009097*Y(3)**2.)
END

```

```

OYSYS SYSTEM NAMELIST
  STIME = 0., FTIME = 10., TSTEP = .05, N = 3,
  Y(1) = 10206., Y(2) = 1739.7, Y(3) = 7200., Y(4) = 1.1807
  IPRINT=1,2,3, PENTC=3., 10., 1.E

```



XMT	=	1.0007	AC	=	1.0212	XNC	=	1.0008
FC	=	5.6120	YWC	=	1.0013	DC	=	0.84012
RE	=	4.3850	CT	=	0.70000	RT	=	1.0005
FFFT	=	0.88000	TBT	=	0.70487	XWT	=	1.0000
TF	=	1226.3	TOC	=	0.312.9	WT	=	100.13
PRC	=	5.0076	DD	=	0.19970			
BC	=	5.0194	CC	=	1.70508E-02			
EEFC	=	0.83823	TRC	=	1.6894			
TI	=	0.88000	ET	=	0.12926			
WC	=	100.13	IC	=	911.73			
TQT	=	14349.						
10.00		5.0000E-02	1.0221E 04		1740.			7205.

EXTREME VALUES OF STATE AND AUXILIARY VARIABLES

	MINIMUM	MAXIMUM
Y( 1)	1.0221E 04	1.0221E 04
Y( 2)	1740.	1741.
Y( 3)	7200.	7205.
Y( 4)	1.181	1.181
Y( 5)	0.6250	0.6260
Y( 6)	0.8058	0.8062

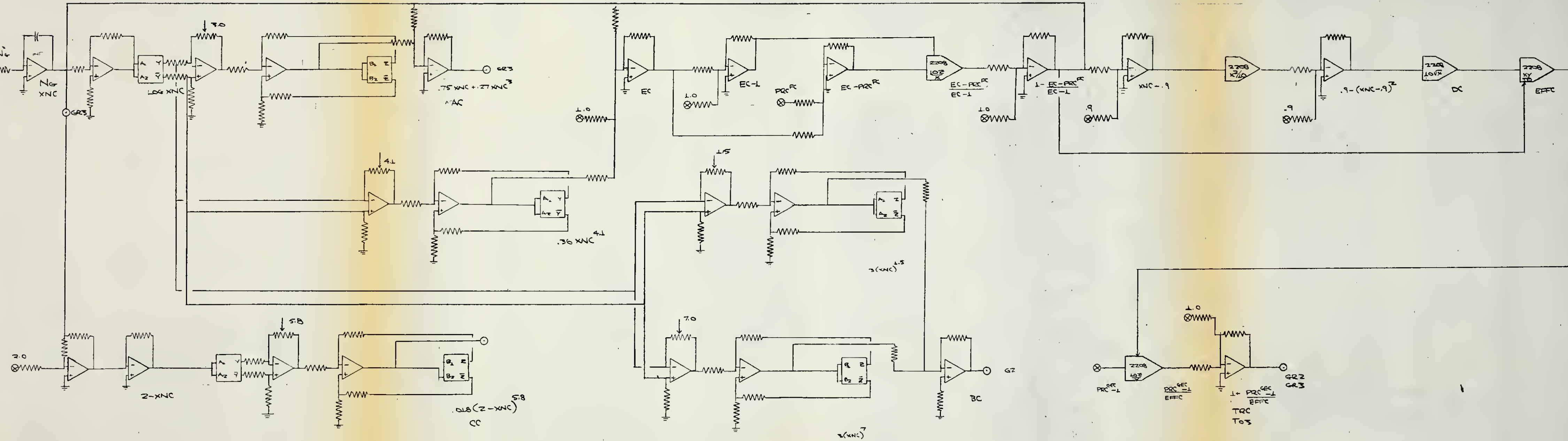


APPENDIX V

DESIGN DRAWINGS AND SKETCHES

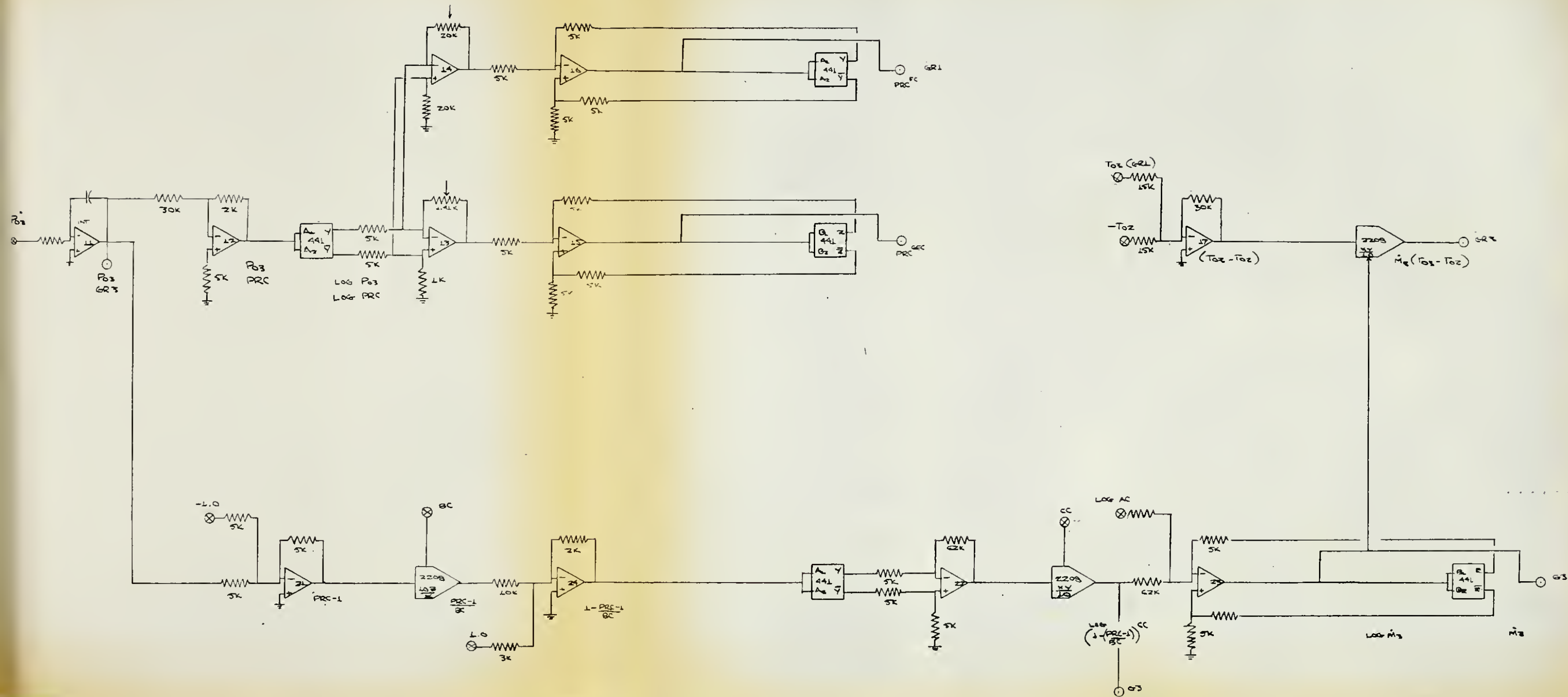




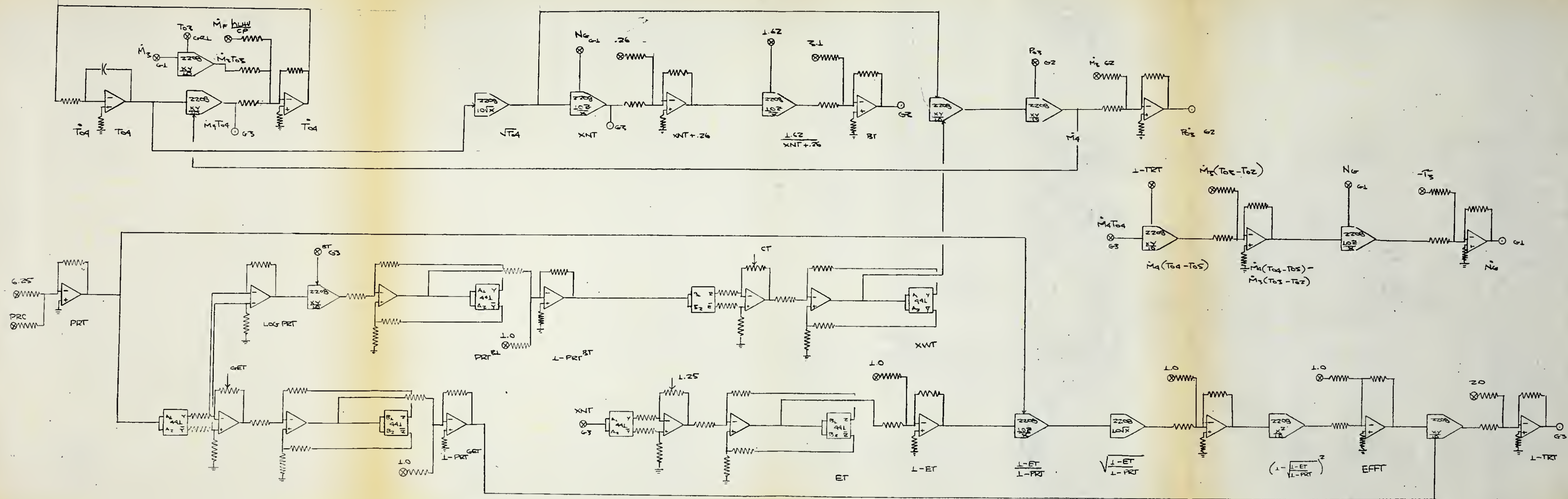




GRID Z





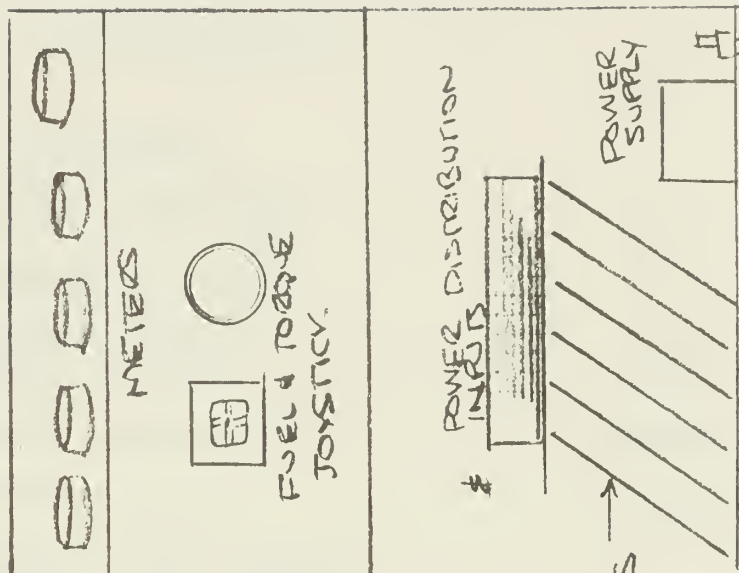




# SIMULATOR LAYOUT

(NOT TO SCALE)

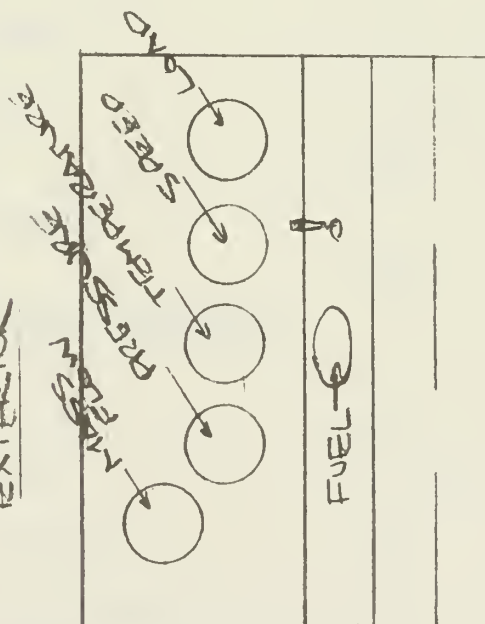
## INTERIOR



HINGE LINE

TOP VIEW

## EXTERIOR



FRONT VIEW





APPENDIX VI  
SPECIFICATIONS

A. ELECTRONIC COMPONENTS

1. Operational Amplifiers

a. Fairchild  $\mu$ A 741

Maximum ratings supply voltage	$\pm 18\text{v}$
Power dissipation	500 MW
Supply current	2.8 MA
Typical input offset voltage	1 mv

b. Fairchild  $\mu$ A 324 Quad OP amp

Maximum ratings supply voltage single polarity	32v
Input voltage range	-0.3v to 32v
Power dissipation	900 MW
Supply current	2 MA
Typical input offset voltage	2 mv

2. EXAR 2208 Four Quadrant Multiplier

Maximum ratings supply voltage	$\pm 18\text{v}$
Power dissipation	750 MW
Typical supply current	5 MA
Typical output nonlinearity	.5%



3. Texas Instruments TL 441 Logarithmic Amplifier

Maximum ratings	
supply voltage	<u>+</u> 8v
Input voltage	6v
Power dissipation	500 MW
Typical differential output offset voltage	40 Mv
Typical supply current $v_{cc}^{+}$	18.5 MA
$v_{cc}^{-}$	8.5 MA

4. Trimming Potentiometers

All applications: Spectrol type 64 ceramic potentiometers, 1-100K $\Omega$

5. Resistors

All applications: Corning 2% 1/4 watt resistors

6. Power Supply

Analog Devices model 975 dual polarity power supply

Output voltage	<u>+</u> 15v
Maximum rating	500 MA
Typical output error	1%

B. PERIPHERALS

1. Breadboard Panels

Continential Specialties model QT 59 6.5" panels



2. Volt Meters

Beede Instrument 3" panel meters,  
model 3-03-8

250<sup>0</sup> movement

200 $\Omega$  internal resistance

3. Joystick

2 - axis movement; two, 100K $\Omega$  rotary  
potentiometers on each axis.

4. Chassis

.062" clear anodized aluminum assembled  
with pop-rivet construction.









Thesis  
S4437  
c.1

Shepherd

The simulation of gas  
turbines by a state of  
the art analog device.

189743

43

gas

Thesis  
S4437  
c.1

Shepherd

The simulation of gas  
turbines by a state of  
the art analog device.

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